The Economics of Skill-Specific Human Capital

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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Para Michelle.

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Andrew Vikram Rendall, Zurich, January 2015

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Part I

Dissertation Overview

Dissertation Overview

Human capital is often expressed in broad terms, allowing for its wide thematic applicability. While economics has mostly avoided such generalizations, it is not exempt from the difficulties in defining human capital, although it is important across a wide range of economic outcomes (e.g., wage formation, marital stability, health). This dissertation focuses on the multi-dimensional nature of human capital skills, and the effect of these skill dimensions on the labor market, using data that has recently allowed for skill-specific definitions of human capital. I am motivated by previous work highlighting the importance of general human capital and skill accumulation. In particular, I introduce multiple human capital skill dimensions to assess the impact of employment tenure on wages in Chapter 1. Chapter 2 focuses on a gender dimension of human capital occupational requirements interacting with life-time wage profiles with respect to career breaks, again focusing on the multi-dimensional aspect of human capital skills. Building upon the ideas presented in the previous chapters, a computable general equilibrium macroeconomic model is developed in Chapter 3 to assess the dynamics between wage inequality and human capital skill dimensions among males. The importance of math skills, and its particular characteristics within the labor market, link together the main results of each chapter. Thus, using micro- and macroeconomic tools, a coherent picture of human capital skill dimensions within the labor market is presented.

In Chapter 1, I include numerical definitions for math, verbal, science and technical skills from the Occupational Information Network (O*net) database within previous definitions of tenure to detail the contribution of specific human capital skills in the observed increasing wage-tenure profile. By merging the O*net information to the National Longitudinal Survey of Youth 1979 (NLSY) sample, I am able to decompose tenure by skill attributes. Previous estimates concentrate on separating the firm, industry and occupation tenure wage effects using industry and occupation codes/labels alone. While I use some similar definitions, I separate the entire career tenure effect into four skill equivalent tenure areas (math, verbal, science and technical). The results indicate that these four skill areas are not uniformly important for all workers. I show that math skills are of primary importance to college educated workers. In contrast, technical skills are shown to be most important for non-college workers, with math and verbal skills somewhat less significant. As in previous studies, I find that employment tenure alone has a negative effect on wages. However, in contrast to previous studies, general career tenure is shown to have a relatively small effect on wages, with virtually all increases in the wage-tenure profile attributable to skill-specific career tenure.

In Chapter 2, co-authored with Michelle Rendall, we present and test the theory that women rationally select occupational paths through preferences for skills that are both resilient and repairable when faced with work gaps. To ground our subsequent analysis, we present a model that generates significant economic incentives for women to strongly prefer occupations that exhibit lower skill-specific depreciation and pursue the accumulation of skills that are robust to work gaps. The examples provided indicate that the combination of skills within an occupation is more important than the occupation itself. That is, if the largest skill component within an occupation is robust to career gaps, then the other skill requirements' atrophy can be offset. Using the NLSY and O*net, we show that college educated women avoid occupations requiring significant math skills potentially due to the costly skill atrophy experienced during a career break. In contrast, verbal skills are very robust to career interruptions. The results support the broadly observed female preference for occupations primarily requiring verbal skills - even though these occupations exhibit lower average wages. Thus, skill-specific atrophy during employment leave and the speed of skill repair upon returning to the labor market are shown to be important factors potentially underpinning women's occupational outcomes. This research suggests that female occupational sorting could be determined by skill-specific atrophy-repair characteristics.

In Chapter 3, co-authored with Michelle Rendall, we are motivated by previous results to investigate a broader math-based structure within a macroeconomic framework that has strong implications for wage inequality. Standard skill-biased technical change (SBTC) is a powerful mechanism in explaining the increasing wage gap between educated and uneducated individuals. However, SBTC cannot explain increasing within-group wage inequality in the US. This chapter provides an explanation for the observed intra-college group inequality by showing that the top decile earners' significant wage growth is underpinned by the link between *ex ante* ability, math-heavy college majors and highly quantitative occupations. We develop a general equilibrium model with multiple education outcomes, where wages are driven by individuals' *ex ante* abilities and acquired math skills. This model emphasizes math-biased technical change (MBTC) and focuses on educational choices. Specifically, individuals make a choice to attempt college or directly enter the labor market. As we are interested in the outcome of the college education process, we directly assign math credits to individuals subject to ability constraints, with some individuals dropping out of college. Math credits characterize each college major in our model. Individuals supply both their *ex post* ability and any acquired math skills to the labor market, with only college graduates supplying the math skills associated with college majors. The results indicate that the SBTC and MBTC mechanisms affect the wage distribution of college graduates in opposite directions. SBTC alone benefits all college graduates in terms of wages, but MBTC is effective at increasing the wages of the top wage deciles of college graduates. Thus, a large portion of within-group and general wage inequality is explained by math-biased technical change (MBTC).

Part II

Chapters

1 The Value of Skill-Specific Experience

1.1 Introduction

The impact of tenure on wages is a seemingly simple positive relationship. However, this superficial correlation conflates a number of important nuances that have driven a five decade long debate covering the wage setting incentives, estimation methods employed, and the measurement of tenure itself. Three main hypotheses have been theorized for the observed increasing wage-tenure profile: better firm-worker matches over time,[1] principle-agent problem/selection screening,[2] and human capital accumulation.[3] Recent literature focuses on human capital accumulation by focusing on three tenure measures, each associated with a specific job dimension: (1) firm, (2) industry; and (3) occupation.

This paper combines data from the National Longitudinal Survey of Youth 1979 (NLSY) sample with the Occupational Information Network (O*net), to assess the importance of skill-specific tenure. Tenure is usually decomposed into a maximum of three components: firm, industry and occupation. However, this means tenure is assessed using labels (e.g., secretary, lawyer), such as the three-digit Standard Industrial Classification (SIC) codes underpinning the US Census industry codes used in the NLSY. In this setup, occupations cannot be meaningfully compared, meaning that the accumulation of skills across occupations cannot be assessed. For example, how similar are accountants and financial analysts? The O*net solves this problem by assigning numerical skill requirements to each occupation across multiple skill dimensions. In this paper, I use broad math, verbal, science and technical skill categories to summarize

[1]See Miller (1984), McCall (1990), Neal (1999) and Gibbons et al. (2005) for discussions concerning the relationship between job search, firm-worker match quality and human capital, including various tenure measurements.

[2]Lazear (1979) and Viscusi (1980) develop models where employee shirking is reduced through deferred compensation. Freeman (1977) and Harris and Holmstrom (1982) suggest that an increasing wage-tenure profile generates insurance-like features for risk-averse workers who are uncertain about future productivity. Salop and Salop (1976), Nickell (1976), Guasch and Weiss (1982) argue that unproductive workers are less likely to apply to firms with increasing wage-tenure profiles. Burdett and Coles (2003) and Stevens (2004) suggest that increasing wage-tenure profiles promote employee retention.

[3]The theoretical link between wage-tenure profiles and firm-specific human capital is well documented by Becker (1962), Oi (1962), Hall (1975), Mortensen (1978), Hashimoto (1981) and Topel (1991).

the skill requirements for each occupation.

Neal (1995) and Parent (2000) find that industry tenure is a primary wage determinant. Specifically, using the NLSY, Parent finds that ten years of industry tenure is associated with an increase in wages of 15 percent, whereas the return to ten years of firm-specific tenure is -9 percent, but not statistically significant. Parent's results using the Panel Study of Income Dynamics (PSID) are similar at 15 percent for industry tenure and -1 percent for firm tenure. Kambourov and Manovskii (2009b) find that industry and firm tenure are of little importance for wages when occupational tenure is included, with ten years of tenure in firm, industry and occupation yielding 1, -2 and 18 percent wage changes, respectively. Sullivan (2010) exploits a coding change in the NLSY starting in 1994 where occupation code changes can reliably used to signal career changes, in contrast to previous efforts that required coding changes across multiple variables to identify a career change. Sullivan finds that both industry and occupation tenure are important determinants of wages for professional, but only occupation tenure matters for non-professionals (e.g., craftsmen). Pavan (2011) notes that the IV estimation method used by Parent, Kambourov and Manovskii and Sullivan, which was first proposed by Altonji and Shakotko (1987) to address the endogenous explanatory variables in such wage regressions, does not account for the two-step employment search process (i.e., career followed by firm), leading to downward biased estimates of firm tenure. Pavan (2011) finds that firm and industry tenure are equally important wage determinants, but later concludes that a meaningful assessment of human capital requires a dataset with information on the tasks and skills used in specific occupations.

This paper precisely expands the research in this dimensions. Specifically, by combining skill information from the O*net with individual-level employment information from the NLSY, I am able to estimate the returns to tenure for specific skills. By decomposing tenure by skill attributes, rather than using industry and occupation labels alone, I show that the accumulation of tenure in math skills is of primary importance to college educated workers. In contrast, technical skills are shown to be most important for non-college workers, with math and verbal skills somewhat less significant. As in previous studies, I find that employment tenure alone has a negative effect on wages. However, in contrast to previous studies, general career tenure is also shown to have a negative effect on wages, with virtually all increases in the wage-tenure profile attributable to skill-specific experience.

The remainder of this paper is organized into three sections. Section 1.2 provides a summary of the data and the basic concepts of the career definitions used in this study. The data analysis, Section 1.3, starts with an explanation of the estimation function and is followed by the results, with a discussion of the different tenure return values for 5 and 10 year time spans. Lastly, Section 1.4 concludes.

1.2 Data

I use two data sources, the National Longitudinal Survey of Youth 1979 (NLSY) and the Occupational Information Network (O*net), versions 4.0-9.0.[4] The NLSY is a individual-level panel data set providing wage, employment, tenure, occupation and career information. The O*net provides numerical skill and task information for all occupations, which is merged with the NLSY. I provide additional detail concerning each data set below.

1.2.1 National Longitudinal Survey of Youth 1979 (NLSY)

The NLSY is a nationally representative sample of individuals aged 14 to 22 in 1979 (i.e., individuals born between 1957 and 1964). Surveys were conducted on an annual basis until 1994 and biannually thereafter. The original sample included 12,686 men and women. I use observations from 1979 to 2000 because the industry coding basis changed starting in 2001.[5]

The NLSY sample provides weekly observations for employment status from which annual employment measures (e.g., tenure) are constructed. I define an individual's main job as the one in which they work the most hours in each week and year. As in Pavan (2011), I restrict the sample to males, and exclude individuals with unobserved initial labor market attachment and those who worked less than 1,200 hours for two consecutive years while employed in a single job of at least 30 hours per week. Similarly, I remove individuals who spend two or more years in military service. Note that full-time workers who return to full-time schooling are included upon reentry into the labor market. In addition, individuals missing important information (e.g., census codes for industry or occupation)[6] and those exhibiting weak labor force attachment[7] are removed. Along the educational dimension, high school dropouts less than 16 years old, high school graduates less than 18 years old and college graduates less than 21

[4] I thank Michelle Rendall for providing access to the NLSY merged O*net/DOT database information.

[5] After 2000, the NLSY industry coding switched from one based on the Standard Industrial Classification (SIC) codes to a new classification system based on the North American Industry Classification System (NAICS). While there are reliable crosswalks across occupation classifications, no equivalent crosswalks exists for industry codes.

[6] Missing census codes for industry and occupation are imputed using a variable linking each job across multiple interview dates, when available.

[7] I follow Pavan (2011)'s weak labor force attachment definition. If an individual drops out of the labor force for only one year, then that observation is deleted. If an individual drops out for more than one year after spending at least ten years in the labor market, that observation and all the subsequent observations are deleted. An individual is completely dropped from the data set if they drop out of the labor force for more than a year after fewer than ten years in the labor market.

Table 1.1: Sample Summary Statistics

VARIABLES	All (1)		LTC (2)		C+ (3)	
Age	27.5	(5.4)	26.9	(5.5)	28.9	(5.0)
Ln Hourly Wage ($1979)	6.2	(0.6)	6.1	(0.5)	6.5	(0.6)
Married	47.8	(50.0)	45.4	(49.8)	54.2	(49.8)
College	27.8	(44.8)				
Experience	8.1	(5.2)	8.4	(5.3)	7.5	(4.9)
Tenure	3.5	(3.5)	3.5	(3.5)	3.7	(3.5)
Career Tenure	4.8	(4.2)	4.6	(4.2)	5.1	(4.2)
Math Tenure	4.8	(4.2)	4.7	(4.2)	5.0	(4.1)
Verbal Tenure	4.7	(4.2)	4.7	(4.2)	4.9	(4.1)
Science Tenure	5.0	(4.3)	4.9	(4.3)	5.1	(4.1)
Technical Tenure	4.9	(4.2)	4.8	(4.3)	5.0	(4.1)
Math Eqv. Tenure	2.3	(2.3)	2.1	(2.1)	2.9	(2.6)
Verbal Eqv. Tenure	2.4	(2.4)	2.2	(2.2)	3.1	(2.7)
Science Eqv. Tenure	2.0	(2.0)	1.9	(1.9)	2.3	(2.1)
Technical Eqv. Tenure	2.3	(2.3)	2.2	(2.3)	2.5	(2.3)
Old Job	62.4	(48.5)	60.8	(48.8)	66.3	(47.3)
Employer Change	26.7	(44.2)	28.3	(45.0)	22.6	(41.8)
Career Change	15.4	(36.1)	17.1	(37.6)	11.0	(31.3)
Math Change	14.0	(34.7)	15.3	(36.0)	10.5	(30.7)
Verbal Change	14.1	(34.8)	15.4	(36.1)	10.8	(31.0)
Science Change	12.7	(33.3)	13.8	(34.5)	9.7	(29.6)
Technical Change	13.5	(34.1)	14.8	(35.5)	10.1	(30.1)
O*net Math	47.4	(17.7)	44.2	(16.9)	55.8	(17.1)
O*net Verbal	50.4	(18.4)	46.5	(17.7)	60.5	(15.9)
O*net Science	40.5	(16.2)	38.5	(15.4)	45.5	(17.1)
O*net Technical	46.6	(19.6)	45.9	(19.4)	48.3	(20.2)
Observations	23,951		17,284		6,667	
Individuals	2,057		1,428		629	

Notes: Standard errors are in parentheses.
Source: NLSY. Males aged 14-22 in 1979. For detailed definitions see text.

years old are also removed from the sample. The total number of individuals remaining in the sample is 2,057 (see Table 1.1).

Annual wages are only recorded every other year since 1994, in contrast to the continuous weekly employment status variables. Thus, approximately 28 percent of wage observations are missing. Unlike Pavan (2011), I exclude all observations with missing wages, which reduces the sample (see Table 1.1). The highest and lowest 10 wage observations are removed as an additional insurance against outliers. Hourly

Table 1.2: Sample Wage Growth Summary: Less than College

VARIABLES	Mean	(Std. Dev.)	N
5-Year Experience	22.9	(54.3)	3,384
10-Year Experience	39.0	(62.3)	2,852
5-Year Firm Tenure	14.7	(40)	2,867
10-Year Firm Tenure	27.9	(43.6)	865
5-Year Career Tenure	18.6	(42.4)	3,365
10-Year Career Tenure	31.7	(49.7)	1,440

Source: NLSY. Males aged 14-22 in 1979. For detailed definitions see text.

wages are deflated using the Consumer Price Index deflator to produce consistent 1979 US dollar values.

Individuals are sorted into two broad educational attainment categories, "LTC" (less than college) and "C+" (college degree or above). Given the time it takes to graduate from college, the college sample is slightly older and has less labor market experience on average. However, years of tenure at a given firm and career tenure are both larger for college graduates. Consequently, the fraction of observations recorded as employer and career changes is smaller for college graduates. I.e., while 17 percent of the non-college graduate observations record a career change, only 11 percent of college graduate observations are classed as career changes.

The usual tenure measures are mostly self-explanatory, such as total labor market experience ("experience"). Additionally, I make use of two tenure measures of skill, a "Skill Tenure" and "Skill Equivalent Tenure," that are explained in more detail in Section 1.2.2. Also related to tenure, "old job" refers to the share of observations that are not new job contracts (first year).

Tables 1.2 and 1.3 summarize the experience tenure wage growth (in percent) for individuals with and without a college degree. As expected, college educated individuals experience considerably faster wage growth than those without college degrees. Both experience and career tenure wage growth are comparable for college graduates, with experience dominating career tenure for non-college graduates. These results highlight the differing labor market characteristics for different education levels. Note, in both cases firm tenure shows the smallest wage growth.

The Armed Services Vocational Aptitude Battery (ASVAB) skill-specific test scores found in the NLSY are used to assess the *ex ante* abilities of individuals for math, verbal, science and technical skills.[8] These tests are based on a set of standardized tests created in WWII by the US military, which were further refined in the mid-1970s by psychometricians who created the first computerized, adaptive tests. Additionally,

[8]The ASVAB was administered in 1980.

Table 1.3: Sample Wage Growth Summary: College

VARIABLES	Mean	(Std. Dev.)	N
5-Year Experience	32.8	(50.5)	1,490
10-Year Experience	55.2	(58.1)	1,058
5-Year Firm Tenure	23.6	(45.4)	1,268
10-Year Firm Tenure	45.6	(46.9)	388
5-Year Career Tenure	30.2	(48.8)	1,470
10-Year Career Tenure	55.2	(57.1)	663

Source: NLSY. Males aged 14-22 in 1979. For detailed definitions see text.

the ASVAB tests' multiple skill dimensions can be turned into composite scores, for career placement purposes. For example, these tests have been used by high schools to assist career counselors. To strengthen the applicability of the ASVAB, the US government had 26 occupational descriptors from the O*net database mapped into seven ASVAB test types by the *ASVAB Career Exploration Program*.[9] Thus, the skill definitions of the ASVAB scores are useful in understanding the human capital skill groups presented in Section 1.2.2. Thus, the seven ASVAB test scores can be mapped into the following four broad categories of skills:

- Math: arithmetic reasoning and mathematics knowledge;
- Verbal: word knowledge and paragraph comprehension;
- Science: general science knowledge; and
- Technical: auto and shop, mechanical comprehension and electronics information.

1.2.2 Occupational Information Network (O*net)

The O*net database contains detailed descriptive information about more than 900 occupations, and succeeds the Dictionary of Occupational Titles (DOT). Whereas the DOT is based on direct expert observations of occupations, the O*net sends questionnaires to a random sample of workers based on their occupations. Each worker completes one-quarter of the questions, which are organized into eight broad categories. Three categories are of particular interest: knowledge, skills and ability, as they provide a natural mapping into math, verbal, technical and science skill groups. This mapping is defined within the O*net and is computed using the database information. These three broad categories cover multiple information areas.

[9] The *ASVAB Career Exploration Program* is sponsored by the Department of Defense, with details on the O*net to ASVAB mapping procedure found in their online technical appendix, www.asvabprogram.com/downloads/Technical_Chapter_2010.pdf.

1. Knowledge: Biology, Building and Construction, Chemistry, Computers and Electronics, Engineering and Technology, English Language, Mathematics, Mechanical, Physics

2. Skills: Equipment Maintenance, Equipment Selection, Installation, Mathematics, Operation and Control, Reading Comprehension, Repairing, Science, Technology Design

3. Ability: Trouble Shooting, Deductive Reasoning, Inductive Reasoning, Information Ordering, Mathematical Reasoning, Number Facility, Oral Comprehension, Written Comprehension

As the four human capital skill groups (math, verbal, science and technical) are based on three broad O*net categories, the O*net does not provide a clean definition of the derived skill groups. However, the O*net created these skill groups to be directly comparable to the ASVAB test scores found in the NLSY (see Section 1.2.1).

The O*net point scales range from zero to five or seven, depending on the category. To ensure consistency and comparability, these values are rescaled to the interval [0,100]. Table 1.1 summarizes the O*net skill measures for the NLSY sample. College graduates tend to work in much higher math and verbal requirement occupations than non-college graduates. Comparing the same groups, the average difference in occupation science requirements is slightly smaller and the difference in terms of technical skills is the smallest, suggesting the latter two could represent more general skills used in unskilled occupations.

The two tenure measures of skill previously mentioned in Section 1.2.1, "Skill Tenure" and "Skill Equivalent Tenure," are used in the empirical analysis below. While the first simply tracks how many years an individual worked in a given skill group (additional details will be provided below), the latter tracks the level of skill used/accumulated over time. For computation of the "skill tenure" variables in Table 1.1 the O*net variables are grouped into 10 equally spaced bins to avoid coding minor skill changes into significant career changes.[10] Thus, an occupation is defined by the four skill types (math, verbal, technical and science), from which they can be further sorted into bins based on the level of each skill required (see Section 1.2.3).

1.2.3 Career Definitions

To accurately capture various dimensions of tenure, I use a similar methodology employed by Parent (2000), Pavan (2011) and Kambourov and Manovskii (2009b), who

[10]Robustness checks with more bins, e.g., 20 bins, yield similar results.

note that spurious occupational changes are observed when only using the occupation census code to define a career change.[11] Thus, I define four tenure variables:

1. Experience: the total labor market tenure for an individual, accounting for employment gaps, defined by the employment status variables in the NLSY.

2. Firm: the total firm-specific tenure for an individual, including possible occupational changes while working within the same firm, defined by the employment status and employer identifier variables in the NLSY.

3. Career: the total occupation-specific tenure for an individual, defined by the employment status, employer identifier, industry and occupation census codes in the NLSY.

4. Career (Skills): the total occupation-specific tenure for an individual, defined by the employment status and employer identifier from the NLSY, combined with a single skill bin change. This, by definition, means that the occupational code must also change, although this is only a necessary condition.

Definitions one through three are identical to previous studies. Category four is only now possible given O*net skill measures. Category four is, by definition, a subset of category three. For example, no occupational change would be recorded under this career definition for an individual switching between two very similar occupations where none of the skill dimensions change substantially. In contrast, an occupational change would be recorded under this career definition for an individual switching between two occupations where the new occupation exhibits a significant change in at least one of the four skill dimensions (e.g., math). If only the math skill moves to a new bins, then a career change in math is recorded, but the other skills would continue as before. This is further illustrated in the summary statistics of Table 1.1 where skill changes are less likely than career changes, with science career changes being the least likely.

I only focus on continuous employment spells,[12] meaning individuals returning to an occupation will reset their occupation-specific human capital levels to zero, whereas Parent (2000) includes human capital accumulation for non-continuous employment spells (i.e., when an individual returns to an occupation, they resume their human capital accumulation from the previous level, rather than from zero). The assumptions underlying continuous employment spells are consistent with the idea of depreciating human capital, proposed by Mincer and Polachek (1974) and further expanded upon by (Rendall and Rendall, 2015, and references therein).

[11] Sullivan (2010) uses a change in the NLSY reporting starting in 1994 to refine his definitions of tenure such that only occupation code changes can be used to signal a career switch.

[12] Results for non-continuous spells are similar in magnitude (slightly larger) and statistical significance to the continuous spell results. Non-continuous spell results are available upon request.

Lastly, experience and tenure measures are yearly observations given the limited wage information in the sample. I follow Kambourov and Manovskii (2009b) in only incrementing tenure variables by one additional year if the individual worked at least 800 hours during the prior calendar year in the given firm/occupation. If the individual works less than 800 hours, the tenure variable remains at the prior year's level until a firm or career change occurs.

1.3 Results

1.3.1 Earnings Function Estimation

To assess the relationship between wages and tenure, I estimate variations of,

$$\ln w_{ifct} = \beta_0 \text{exp}_{it} + \beta_1 \text{OJ}_{ift} + \beta_2 \text{ften}_{ift} + \beta_3 \text{cten}_{ict} + \theta_{it} \, . \tag{1.1}$$

On the left-hand side, $\ln w_{ifct}$ is the natural log of hourly wages ($1979) for individual i in period t, with firm f and career c. On the right-hand side, exp is total labor market experience, OJ ("old job") is a dummy variable equal to one when firm tenure is greater than one, ften is firm tenure and cten is career tenure. OJ is included to account for any non-linear effect of firm-specific tenure during the first year, such as an initially high rate of on-the-job learning. In addition, any non-linearity in experience and tenure will additionally by captured by squared and cubed terms of the exp, ften, and cten variables. Variables that capture specific occupational characteristics, such as industry and occupation codes, are also included, but vary by regression specification and are discussed in-detail in Section 1.3.2. Other variables in Equation (1.1) includes an intercept term, a union dummy, marital status dummies,[13] year dummies, region dummies, education dummies,[14] race dummy,[15] and four ASVAB skill-specific test scores (math, verbal, science and technical) as a measure of innate ability.[16]

The variables that are not included in Equation (1.1) are either unobserved or not cleanly reported. These include individual-specific charateristics, such as motivation,

[13]marital status dummies are: married, never married and other

[14]Degree dummies are: high school dropout, high school graduate, some college, college graduate and post-graduate degree.

[15]Dummy variable equals one if an individual is black.

[16]The ASVAB skill-specific test scores are reported for each individual. Previous studies do not include an innate ability measure, as they explicitly state that such a measure is a fixed effect contained in the error term (e.g., Kambourov and Manovskii, 2009b). The results presented in this paper are consistent with this interpretation, with very similar results when including and excluding the ASVAB scores. Results without the ASVAB scores are available upon request.

and firm-/career-worker match quality measurements. Thus,

$$\theta_{it} = \alpha_i + \mu_{if} + \gamma_{ic} + \epsilon_{it} \ , \tag{1.2}$$

where α_i is the individual effect, μ_{if} is the firm-worker match effect, γ_{ic} is the career-worker match effect and ϵ_{it} is the usual error term.

Proceeding with Equation (1.1) using ordinary least squares (OLS) is problematic because it is probable that unobserved components are correlated with the observed variables, such as tenure and experience. For example, individuals with large α_i terms might be favored in the labor market and thus exhibit higher quality firm- and career-worker matches. Similarly, higher μ_{if} and γ_{ic} may indicate better employment matches, leading to longer tenure and higher wages. I address this endogenity issue using the instrumental variables (IV) approach developed by Altonji and Shakotko (1987).[17] Specifically looking at one career spell for individual i with tenure X_{ict}, I calculate the average tenure during that career spell as $\overline{X}_{ic}$. Thus, the instrumental variable for tenure can be computed as, $\hat{X}_{ict} = X_{ict} - \overline{X}_{ic}$, with the squared and cubed tenure instrumental variables similarly computed. By construction, these IVs are orthogonal to the career-match component, i.e., the endogeneity between wages and career-worker match quality is removed. For completeness, I also instrument firm tenure, labor force experience and the OJ dummy using their respective spell-specific means. Lastly, as the NLSY is a panel dataset, I estimate the instrumented model using the generalized least squares (GLS) approach to account for the serial correlation of individual error terms. Results are provided for both OLS and IV-GLS for comparison purposes, although the IV-GLS results are preferred for the reasons detailed above.

1.3.2 Empirical Results

Three regression specifications are estimated using Equation (1.1) as a basis, each differing in terms of career definition.

1. This specification is the basic regression from the literature, where career tenure is defined by the NLSY 3-digit occupation and industry census codes.

2. Using the basic regression from specification 1, I make a simple change to the career tenure definition to include the previously defined four O*net human capital skills (math, verbal, science and technical). This specification allocates the human capital skills for each occupation into 10 equally spaced bins from lowest skill requirements to highest. When an individual enters a new occupation, the

[17] This approach is also used by Parent (2000) and Kambourov and Manovskii (2009b), with Sullivan (2010) using a similar methodology.

bins for the four skills for the old and new occupation are compared. Career changes are tracked by skills separately. A bin change for any skill is registered as a skill-specific career change. For example, assume an individual works in occupation A, with skill percentile ranks of 35, 49, 71 and 44 percent for math, verbal, science and technical skills, respectively. The career tenure identification uses the following bins, respectively: 4, 5, 8 and 5. Now assume this individual's occupation code changes to occupation B with the following skill percentile ranks, respectively: 39, 49, 79 and 41. As the bins for occupation A and B are identical, no career change is registered for any skill. Now assume this individual switches from occupation B to C with the following skill percentile ranks, respectively: 41, 49, 79 and 39. This switch will trigger a career change in math, as the math bin changed from 4 to 5. It will also trigger a career change in technical skills, as the individual moves from bin 5 to 4. However, no career change will be recorded for verbal and science skills. The career tenure measure continues to track years in this specification. That is, two individuals that work at least 800 hours in their respective math bins 10 consecutive years, will have both accumulated 10 years of math tenure, even though one may be working in bin 1 and the other in bin 5.

3. To further refine the idea of skill-specific tenure, specification 3 accounts for the accumulation of specific skills based on both the length of occupational employment and the skill level used. Using the same NLSY 3-digit occupation and industry codes to define career tenure from specification 1, I add four skill equivalent tenure measures, defined as: $\text{sten}_{ic} = \sum_{\tau=1}^{t} \text{skill}_{ict} \times 1_{(hours_\tau \geq 800)}$, for math, verbal, science and technical skills in current career c. For example, the math skill equivalent tenure measure for an individual employed three years in career A, which uses a 0.35 math skill level for the first two years and then (after a promotion) uses 0.4, would be: $1.1 = (1 \times 0.35) + (1 \times 0.35) + (1 \times 0.4)$. That is, even though the math content may change, it is not considered a new career as it happens within the firm (see definition 1), which is consistent with career changes in the current literature. This measure simply addresses the idea that career tenure might have different importance for different levels of skills (Sullivan, 2010).

Thus, the three specifications can be thought of as: (1) the standard regression specification from the literature; (2) a simple revision of the standard regression specification from the literature to include human capital skill types; and (3) the basic regression from the literature, but with an interacted human capital skill experience measure. Note that the results for specification 2 and 3 share the same human capital skill labels, although they have different definitions.

Table 1.4: OLS Estimation Results

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Experience	0.041***	(0.006)	0.042***	(0.007)	0.038***	(0.006)
Experience^2 × 100	-0.220***	(0.068)	-0.227***	(0.072)	-0.209***	(0.069)
Experience^3 × 100	0.004*	(0.002)	0.004*	(0.002)	0.004*	(0.002)
Old Job	0.000	(0.010)	-0.001	(0.011)	0.002	(0.010)
Firm Tenure	0.007	(0.004)	0.012***	(0.005)	-0.000	(0.005)
Firm Tenure^2 × 100	-0.042	(0.027)	-0.071**	(0.029)	-0.037	(0.028)
Career Tenure	0.077***	(0.006)			0.073***	(0.007)
Career Tenure^2 × 100	-0.530***	(0.076)			-0.574***	(0.087)
Career Tenure^3 × 100	0.014***	(0.003)			0.016***	(0.003)
M Tenure			0.013	(0.013)	0.003	(0.019)
M Tenure^2 × 100			0.009	(0.168)	0.277	(0.389)
M Tenure^3 × 100			-0.002	(0.006)	-0.011	(0.021)
V Tenure			0.028**	(0.012)	-0.033*	(0.017)
V Tenure^2 × 100			-0.325**	(0.158)	0.581*	(0.326)
V Tenure^3 × 100			0.009	(0.006)	-0.026	(0.017)
S Tenure			0.010	(0.012)	0.042**	(0.021)
S Tenure^2 × 100			-0.042	(0.156)	-0.773	(0.497)
S Tenure^3 × 100			0.001	(0.006)	0.035	(0.032)
T Tenure			0.022**	(0.011)	0.036**	(0.014)
T Tenure^2 × 100			-0.155	(0.146)	-0.382	(0.272)
T Tenure^3 × 100			0.006	(0.005)	0.013	(0.014)
Observations	24,091		24,049		23,929	
R-squared	0.367		0.347		0.359	

Statistical significance levels: *** p<0.01, ** p<0.05, * p<0.1

Notes: Robust standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

All regressions include dummies for years, 1-digit industry dummies, race (Black or Other), region, marital status (married, never married, and other), union membership, and last school degree (high school drop out, high school graduate, some college, college graduate, and post-college graduate). Regression (1) also includes 1-digit occupation dummies, while regressions (2) and (3) include the four O*net skill requirements by occupation (math, verbal, science, and technical).

Table 1.4 provides OLS estimates for all regression specifications. Looking at the results for specification 1, both labor market experience and career tenure are important and statistically significant at the one percent level. Specification 2 gives nearly identical results for experience, but now attributes the remaining wage effects to verbal and, to a lesser degree, technical skills. Specification 3 matches the experience and career tenure estimates found in specification 1, but now highlights science, technical

Table 1.5: IV-GLS Estimation Results

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Experience	0.090***	(0.007)	0.087***	(0.007)	0.179***	(0.018)
Experience^2 $\times$ 100	-0.417***	(0.055)	-0.371***	(0.057)	-0.356***	(0.054)
Experience^3 $\times$ 100	0.008***	(0.002)	0.007***	(0.002)	0.007***	(0.002)
Old Job	-0.028***	(0.009)	-0.034***	(0.010)	-0.031***	(0.009)
Firm Tenure	-0.012***	(0.004)	-0.013***	(0.004)	-0.028***	(0.004)
Firm Tenure^2 $\times$ 100	0.009	(0.024)	0.011	(0.025)	0.055**	(0.025)
Career Tenure	0.044***	(0.006)			0.000	(0.008)
Career Tenure^2 $\times$ 100	-0.343***	(0.068)			-0.098	(0.082)
Career Tenure^3 $\times$ 100	0.009***	(0.002)			0.004	(0.003)
M Tenure			0.015	(0.011)	0.051***	(0.016)
M Tenure^2 $\times$ 100			-0.068	(0.133)	-0.512	(0.331)
M Tenure^3 $\times$ 100			0.000	(0.005)	0.027	(0.019)
V Tenure			0.016	(0.011)	0.025*	(0.015)
V Tenure^2 $\times$ 100			-0.198	(0.133)	-0.010	(0.289)
V Tenure^3 $\times$ 100			0.007	(0.005)	-0.002	(0.016)
S Tenure			0.020*	(0.011)	0.033*	(0.017)
S Tenure^2 $\times$ 100			-0.143	(0.131)	-0.780**	(0.386)
S Tenure^3 $\times$ 100			0.002	(0.005)	0.029	(0.024)
T Tenure			0.005	(0.010)	0.055***	(0.013)
T Tenure^2 $\times$ 100			-0.053	(0.129)	-0.594***	(0.227)
T Tenure^3 $\times$ 100			0.004	(0.005)	0.020*	(0.012)
Observations	24,091		24,049		23,929	
Individuals	2,052		2,052		2,048	

Statistical significance levels: *** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See notes Table 1.4 and text for details.

and verbal skill equivalent tenure as the drivers of wages. The estimates indicate that firm tenure is not a particularly important wage determinant, and is only statistically significant under specification 2. Note that these results are broadly consistent with the previous literature. For example, Kambourov and Manovskii (2009b) also find relatively large and statistically significant results for labor market experience under OLS, despite using different underlying data. However, it is important to note that these OLS results are provided as a link to previous research results, and are not indicative of the actual relationship between wage and tenure measures for reasons detailed in Section 1.3.1.

Table 1.5 provides IV-GLS estimates for all regression specifications. Reviewing the coefficient estimates for specification 1 shows that experience, firm and career tenure all have large and statistically significant wage effects, although the firm tenure wage effect is negative. This result is similar Sullivan (2010) who finds a negative firm and large positive occupation tenure effect for professional-level workers, both being statistically significant. Specification 2 reduces the relative impact of labor market experience and finds relatively small and statistically insignificant coefficients on the human capital skill measures. Specification 3 produces the strongest results across the board, with experience, firm tenure, math and technical skill equivalent tenure having a substantial and statistically significant effect on wages. The robust results of specification 3 are intuitive, given that the skill equivalent tenure measures more closely align with the definition of skill accumulation. I.e., the longer an individual works in a particular occupation, the more skill-specific experience they accumulate. However, the accumulation of tenure (or learning-by-doing) matters more the higher the skill content of the job (See Sullivan, 2010). For example, working in the lowest math occupation (skill level of zero) would add nothing in terms of wages, even with more tenure, while working in the highest math occupation (skill level one) would make math tenure highly important for wage growth over the life-cycle. Similarly to the OLS results presented, many of the IV-GLS results are broadly consistent with the relevant NLSY-based IV-GLS estimates found in the literature. For example, Parent (2000) finds a one year labor market experience wage effect of about 13 percent, compared to 18 percent in specification 3 of this study. However, the results of this study diverge considerably from previous work when skill measures are included, with large, statistically significant coefficients for both math and technical skills under specification 3. In comparison, both verbal and science skills exhibit much smaller coefficients that are less statistically significant.

1.3.3 Discussion

Since the returns to tenure are non-linear, I provide returns to tenure for 2, 5 and 10 years for three sample groups: all individuals (Table 1.6), individuals without a college degree (Table 1.7) and individuals with a college degree (Table 1.8). Table 1.6 uses the IV-GLS results presented in Section 1.3.2, whereas Tables 1.7 and 1.8 are based on separate IV-GLS estimates for each sub-sample (see Appendix A.1).

Looking at the results for all individuals (Table 1.6) under specification 1, five years of firm tenure leads to a decrease in wages of 8.5 percent. Sullivan (2010) calculates a similar five year value for firm tenure at -6 percent under a similar specification, but

Kambourov and Manovskii (2009b) find firm tenure to have neglible wage effects.[18] Counteracting this is a positive career tenure value, with five years of career tenure associated with 15 percent higher wages. Sullivan calculates about 18 percent wage growth for the same career tenure. Specification 2 exhibits similar returns to firm tenure as specification 1, but identifies math and science skill tenure as the primary wage drivers. However, specification 2 does not provide the statistical significance required to fully commit to the estimated returns for skill tenure. Specification 3 points to a significant negative effect for both firm and career tenure, with 10 years of tenure leading to a reduction of average wages by 26 and 6 percent for specification 3, respectively. The specification 3 skill equivalent tenure returns assume an individual working in the highest (100th percentile) occupation for the specified skill. For example, an individual working in an occupation with the highest math requirement (e.g., physicist) will see a wage increase of 27 percent over ten years, *ceteris paribus*. Alternatively, an individual working in an occupation at the 50th math percentile will experience wage growth over 10 years that is equivalent to the 5 year tenure returns reported (i.e., 16 percent). For comparison, an individual working in the bottom math skill occupation will experience no wage growth related to tenure within that career path. Thus, math, verbal and technical skill equivalent tenure push wages significantly higher. Since Table 1.1 suggests that non-college graduates and college-graduates select into occupations with very different skill requirements, I further explore skill-specific tenure returns using these two broad educational categories. In order to compute education-specific returns, IV-GLS estimates were produced for both college and non-college sub-samples. The returns to tenure by education group are reported and discussed below.

Investigating the returns to tenure for individuals without a college degree requires estimating Equation (1.1) using IV-GLS using the relevant sub-sample of the NLSY. The calculated returns from this process are shown in Table 1.7. Specification 1 shows a similar negative effect for firm tenure as the full NLSY sample, with a somewhat less positive estimate for career tenure. Specification 2 focuses attention on math and science skills, although neither are statistically significant. Specification 3 depicts the most interesting results for those without a college degree. As with the full NLSY sample, firm and career tenure have a strong negative and statistically significant impact. However, in contrast to the full NLSY sample, technical and verbal skill equivalent tenure have the strongest positive impact on wages. Math continues to be an important positive determinant of wages, but science is not statistically significant, although it exhibits a large negative impact (i.e., -27 percent for 10 years of science skill equiv-

[18] Kambourov and Manovskii (2009b) reports firm tenure wage effects similar to those of Altonji and Williams (2005) (i.e., small/neglible), who attempt to reconcile the differing firm tenure wage effects reported in Altonji and Shakotko (1987) and Topel (1991).

Table 1.6: IV-GLS Returns to Tenure, All Individuals

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Firm: 2 years	-0.052***	(0.009)	-0.059***	(0.009)	-0.085***	(0.010)
Firm: 5 years	-0.085***	(0.014)	-0.094***	(0.015)	-0.157***	(0.016)
Firm: 10 years	-0.137***	(0.020)	-0.149***	(0.022)	-0.255***	(0.023)
Career: 2 years	0.075***	(0.010)			-0.004	(0.013)
Career: 5 years	0.147***	(0.017)			-0.020	(0.023)
Career: 10 years	0.191***	(0.020)			-0.061*	(0.029)
Math: 2 years			0.027	(0.017)	0.084***	(0.022)
Math: 5 years			0.058*	(0.029)	0.162***	(0.030)
Math: 10 years			0.084*	(0.033)	0.271***	(0.043)
Verbal: 2 years			0.025	(0.017)	0.050*	(0.020)
Verbal: 5 years			0.041	(0.030)	0.122***	(0.030)
Verbal: 10 years			0.035	(0.034)	0.226***	(0.044)
Science: 2 years			0.035*	(0.017)	0.037	(0.022)
Science: 5 years			0.069*	(0.029)	0.006	(0.031)
Science: 10 years			0.078*	(0.034)	-0.158**	(0.060)
Technical: 2 years			0.008	(0.016)	0.088***	(0.018)
Technical: 5 years			0.015	(0.028)	0.152***	(0.028)
Technical: 10 years			0.031	(0.032)	0.157***	(0.039)

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Standard errors in parentheses.

Returns to tenure are computed from regression results Table 1.5.

alent tenure). Therefore, for individuals without a college degree, a career in a highly technical or highly verbal occupation generates the largest possible wage gains over their working life.

The returns to tenure presented in Table 1.8 come from the same process as those in Table 1.7, except the NLSY sub-sample contains those with college degrees only. Under specification 1, firm tenure's estimated 10 year effect is -17 percent, whereas the equivalent career tenure effect is 27 percent - offsetting the negative firm tenure effect. Specification 2 links the similar negative effect of firm tenure with relatively large math and science skill effects, at 14 and 17 percent for 10 year tenure, respectively. However, these effects are only statistically significant at 10 percent. Turning to specification 3, we see that math skill equivalent tenure returns are very large for college graduates, e.g., 25 percent for 10 years tenure at the 100th math percentile occupation. This relationship is statistically significant at the one percent level and is significantly larger than the other skills. Science skill equivalent tenure return for 10 years employment in a 100th percentile science occupation is 22 percent, but the statistical significance

Table 1.7: IV-GLS Returns to Tenure, Less Than College

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Firm: 2 years	-0.047***	(0.010)	-0.050***	(0.011)	-0.078***	(0.012)
Firm: 5 years	-0.075***	(0.016)	-0.081***	(0.018)	-0.144***	(0.019)
Firm: 10 years	-0.122***	(0.023)	-0.133***	(0.025)	-0.237***	(0.027)
Career: 2 years	0.060***	(0.011)			-0.013	(0.014)
Career: 5 years	0.114***	(0.019)			-0.038	(0.026)
Career: 10 years	0.143***	(0.023)			-0.082*	(0.033)
Math: 2 years			0.021	(0.019)	0.094***	(0.026)
Math: 5 years			0.044	(0.033)	0.145***	(0.036)
Math: 10 years			0.055	(0.038)	0.186**	(0.059)
Verbal: 2 years			0.017	(0.019)	0.029	(0.024)
Verbal: 5 years			0.028	(0.033)	0.108**	(0.035)
Verbal: 10 years			0.027	(0.038)	0.276***	(0.058)
Science: 2 years			0.036	(0.019)	0.052*	(0.025)
Science: 5 years			0.063	(0.032)	-0.002	(0.036)
Science: 10 years			0.055	(0.037)	-0.269***	(0.077)
Technical: 2 years			0.000	(0.018)	0.084***	(0.020)
Technical: 5 years			0.008	(0.031)	0.178***	(0.031)
Technical: 10 years			0.043	(0.036)	0.280***	(0.046)

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Standard errors in parentheses.

NLSY (males aged 14-22 in 1979). Returns to tenure are computed from regression results Table A.1.

only reaches the 10 percent level. Verbal skills seem to matter, but only in the first few years before the effect vanishes, while technical skills have no wage returns to tenure.

The main effects of including human capital skill tenure returns can be easily seen when viewing the results separately for college and non-college workers. Concentrating on specification 3, as this specification provides the most intuitive skill accumulation definition, there are significant differences between college and non-college education groups' tenure returns. While both groups see significant wage decreases with firm tenure, they experience wage growth through different skill channels. Specifically, non-college workers benefit most from skill equivalent tenure in technical and verbal human capital skill areas, and to a lesser extent math. This result diverges from the results for college graduates, where math and science skill equivalent tenure yields the largest positive wage effect. For college graduates, technical skill equivalent tenure yields no wage effect, with verbal skills becoming insignificant over time, both in terms of size and statistical significance.

Table 1.8: IV-GLS Returns to Tenure, College

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Firm: 2 years	-0.062***	(0.016)	-0.080***	(0.018)	-0.092***	(0.018)
Firm: 5 years	-0.105***	(0.025)	-0.129***	(0.028)	-0.162***	(0.029)
Firm: 10 years	-0.166***	(0.035)	-0.190***	(0.039)	-0.249***	(0.041)
Career: 2 years	0.120***	(0.019)			0.017	(0.029)
Career: 5 years	0.227***	(0.033)			0.030	(0.052)
Career: 10 years	0.267***	(0.039)			0.023	(0.063)
Math: 2 years			0.043	(0.033)	0.082*	(0.041)
Math: 5 years			0.090	(0.056)	0.160**	(0.059)
Math: 10 years			0.137*	(0.062)	0.254***	(0.072)
Verbal: 2 years			0.043	(0.036)	0.108**	(0.042)
Verbal: 5 years			0.068	(0.061)	0.126*	(0.062)
Verbal: 10 years			0.031	(0.071)	0.020	(0.078)
Science: 2 years			0.038	(0.037)	0.011	(0.047)
Science: 5 years			0.089	(0.062)	0.083	(0.068)
Science: 10 years			0.166*	(0.070)	0.215*	(0.110)
Technical: 2 years			0.034	(0.035)	0.059	(0.041)
Technical: 5 years			0.044	(0.058)	0.063	(0.062)
Technical: 10 years			-0.009	(0.064)	-0.093	(0.085)

Statistical significance levels: *** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses.

Returns to tenure are computed from regression results Table A.2.

1.4 Conclusion

Much of the previous literature linking tenure and wages concentrated on three areas of advancement: understanding the wage setting incentives, the estimation methods employed, and the measurement of tenure itself. This paper moves this literature forward by including nuanced numerical measures of human capital skill accumulation covering math, verbal, science and technical skills within the framework employed by a large part of the existing literature relevant to the topic, using the favored estimation methodology.

The results redefine the wage-tenure relationship along one significant dimension: careers. Previous estimates concentrate on separating the firm, industry and occupation tenure wage effects. While I use some similar definitions, I can separate the entire career tenure effect into four skill equivalent tenure areas. These four skill areas are not uniformly important for all workers. Workers without a college degree experience the largest wage growth through the accumulation of technical and verbal skill equivalent tenure, with math being of third-order importance. College graduates benefit most from accumulating math and science skill equivalent tenure, but experience virtually no effect from verbal skill equivalent tenure.

The current analysis follows the strict definition of career changes previously employed in the literature. I do this in order to use similar samples (time periods and individuals) to estimate the newly defined skill-career tenure effects and make the results comparable to the broader literature. However, as Sullivan (2010) points out, a change in the variable collection process of the NLSY in 1994 now allows for the identification of career changes within a given firm. In future research I plan on extending the current set of results to account, not only for the effects of skill-tenure when moving from employer to employer, but also the effect of skill-tenure when staying with a current employer.

2 Women and Careers: Skill-Specific Atrophy and Repair

Joint with Michelle Rendall

2.1 Introduction

The gender wage gap is a persistent characteristic of the US labor market. Although it has narrowed significantly between 1970 and 2010, research suggests that it will not disappear for a number of reasons. Favored explanations generally focus on female-male differences in bargaining, fertility, and preferences across occupations (see Goldin, 2014, and references therein). In this paper, we focus on the idea that women and men have different economic valuations (preferences) over human capital types, which is underpinned by their employment expectations. The subsequent specific capital accumulation leads to an occupation gap between men and women that is surprisingly robust.[1]

The precise mechanism underpinning the differences between female and male occupational choices is still under debate. The traditional model (Roy, 1951) suggests that individuals choose occupations that maximize their skill returns. However, this model cannot explain the large labor market gender differences observed, including occupational choices over the life-cycle. Thus, the basic question is: How do women (men) choose occupations?

Hsieh et al. (2013) provides detailed statistics underlining the observed female-male occupational differences, along with the closing (but still existing) gender gap in occupational choice. The authors argue that the misallocation of talent from 1960 to today has shrunk, as frictions in both the labor market and schooling choices have decreased. They model both frictions as taxes that diminish over time based on changes in occupational barriers, the distribution of talent and occupation-specific technical change. However, even with a decrease in these frictions, the occupational gender gap is still significant today. More specifically, the occupation similarity index (see Table 1 in Hsieh et al., 2013), where zero denotes no overlap and one denotes a perfect overlap

[1] Although occupational differences across gender have diminished rapidly, the pace of convergence has slowed recently.

with the occupational distribution of white men, increases from 0.38 to 0.59 from 1980 to 2008 for higher educated white women and from 0.40 to 0.46 for lower educated women. Thus, while men and women have similar *ex ante* abilities (Goldin et al., 2006), women self-select into vastly different occupations compared to men. Goldin (2014) suggests that the penalty attributed to part time work or the inflexibility of work schedules of certain occupations is a primary driver of occupational differences. In the context of Hsieh et al. (2013), rigid work schedules are a friction that has yet to be overcome.

We propose a mechanism that is complimentary to Hsieh et al. (2013) and Goldin (2014), and consistent with the large skill-biased technical change literature. Technological innovation in the last few decades has been exceptionally fast by historical standards, especially within the ICT sector. Some skills may be more exposed to this innovation than others. That is, as technology moves forward, certain skills may become obsolete more quickly. If women are more likely to take career breaks, e.g., for child bearing/rearing, they may optimally choose occupations that exhibit less skill-obsolescences if they experience work gaps. Therefore, we explore if certain types of skills are more likely to become obsolete in the labor market after career breaks.

This idea follows from the large literature explaining occupational choices through human capital characteristics. The gender dimension is first explored by Mincer and Polachek (1974) who theorize that women acquire human capital taking into account their expectations regarding family formation and future labor market attachment. The authors estimate human capital depreciation rates for women from the National Longitudinal Survey of Mature and Young Women (NLS). Polachek (1981) takes the generalized depreciating human capital concept and allows for occupation-specific skill depreciation, with the author concluding that occupational choice is related to the period of time spent out of the labor force. McDowell (1982) similarly notes that women tend to avoid fields where knowledge depreciates quickly (is non-durable) and this selection bias is correlated with aggregate fertility patterns. Mincer and Ofek (1982) find evidence of wage "rebound" when estimating income losses from labor market withdrawal and re-entry. They hypothesize that this wage rebound is actually a form of "repairing" or relearning previously depreciated human capital, based on the assumption that relearning skills is less costly than learning a task for the first time. Additional support comes from Lazear and Rosen (1990), who suggest women are passed up for promotions within the same "narrow" jobs due to a lack of firm-specific human capital, possibly due to career interruptions.

The previous literature, due to a lack of data availability, used occupation labels (e.g., lawyer, nurse) or the share of women within an occupation to differentiate between male and female human capital types.[2] Given new data sources, this study focuses on gender differences in the demand and supply of specific skills such as mathematics and language. That is, we expand on the literature by analyzing whether women choose certain occupations because of skill-specific atrophy and repair rates. Although *ex ante* women possess similar abilities to males, women will potentially prefer lower paying occupations when maximizing expected lifetime income because the skills required in these occupations have small absolute depreciation rates.[3] Depreciation rates are especially important for women who expect employment gaps (e.g., child-rearing). For example, Adda et al. (2012) build a model where fertility affects career paths through initial occupational decisions. The authors study how German women make career choices within the apprenticeship system, given that these women will make fertility choices during their working years. The difference between our work and Adda et al. (2012) is two-fold. First, we focus on occupation decisions related to certain skills (i.e., math, verbal, science, technical skills), both for college graduates and non-college workers. Second, we consider occupational choices throughout an individual's life-cycle. Therefore, the goal of this paper is to quantitatively assess whether there is a gender bias in occupational choices based on skill requirements and to what extent varying skill-specific atrophy-repair rates exist.

Section 2.2 starts with a short summary of the data and the basic facts concerning occupational gender differences in the National Longitudinal Survey of Youth 1979 (NLSY) sample. Section 2.3 provides a simple model of individual occupational choice. We use the model to derive a monetized mismatch of skill measure, based on occupation requirements by gender-education groups and absolute depreciation rates. The data analysis, Section 2.4, has two main objectives. First, we detail the gender-specific relationships between *ex ante* ability and *ex post* occupation outcomes. We compute the mismatch of men and women by skill type from the NLSY. Second, we document skill-specific atrophy-repair functions from the NLSY. The NLSY is ideally suited to compute mismatch and atrophy-repair rates, as it provides individual ability measures (e.g., math) through the Armed Services Vocational Aptitude Battery (ASVAB), along with detailed work histories. In conjunction with the NLSY, the Occupational Information Network (O*net) provides the necessary occupation-specific skill measures. Section 2.5 concludes.

[2] An occupation is classified to have female human capital if the share of female workers within that occupation surpasses a certain threshold.

[3] In this study, absolute depreciation rates are defined as the depreciation of skill taking into account potential repair rates when reentering the labor market.

2.2 Skills and Occupations

We make use of two data sources, the NLSY (National Longitudinal Survey of Youth 1979) and the O*net (Occupational Information Network) versions 4.0-9.0. These datasets provide two unique descriptive dimensions for occupations: (1) individual skills and wage returns to various skills in each occupation; and (2) O*net descriptors, where occupations are differentiated by the tasks and skills they require, rather than the *ex ante* abilities of individuals within those occupations.

To assess the *ex ante* abilities of individuals, we use the NLSY, which records skill-specific test scores for math, verbal, science and technical skills from the ASVAB administered in 1980 (see Appendices B.1 and B.2 for details on the data). These tests are based on a set of standardized tests created in WWII by the US military, which were further refined in the mid-1970s by psychometricians who created the first computerized, adaptive tests. The ASVAB tests multiple skill dimensions, turned into composite scores, for career placement purposes. These tests are commonly used by high schools to assist career counselors. The NLSY cohort was tested using the 1980 version of these exams, with results for each individual providing relative skill measures. In addition, 26 occupational descriptors from O*net have been mapped into seven ASVAB test types by the *ASVAB Career Exploration Program*.[4]

Table 2.1 summarizes the data used in the analysis using broad education groups.[5] Although individuals were interviewed from 1979 to 2010, the sample here only includes observations after individuals graduated from their highest degree (i.e., all students are dropped). The empirical analysis differentiates wages observations of part-time and full-time workers., where part-time workers are individuals that worked at least 500 hours but no more than 1,400 hours in a calendar year.

In the empirical analysis we use two samples. One including all individuals with valid occupational observations and wages, the other only including workers with substantial labor force attachment. The labor force attachment variable in Table 2.1 computes the share of individuals that spend at least 75 percent of their life-cycle (after graduating) working. Not surprisingly, this share is considerably higher for men than women, and also larger for college graduates compared to non-college graduates. Consistently, summary statistics on the total time spent at home (either as unemployed or not in the labor force) are generally higher for women than men and for non-college than college-graduates. Moreover, women not only have a higher mean number of week

[4]The *ASVAB Career Exploration Program* is sponsored by the Department of Defense; more details on the mapping procedure can be found at www.asvabprogram.com/downloads/Technical_Chapter_2010.pdf.

[5]"LTC" denotes individuals without a college degree, and "C+" denotes individuals that have completed at least a four-year college degree.

Table 2.1: Sample Summary Statistics

	Male				Female			
VARIABLES	LTC (1)		C+ (2)		LTC (3)		C+ (4)	
Year	1,992	(8)	1,995	(8)	1,992	(9)	1,995	(8)
Age	31	(9)	35	(8)	32	(9)	35	(8)
Married	46	(50)	60	(49)	52	(50)	59	(49)
Graduation Year	1,979	(4)	1,985	(4)	1,980	(4)	1,985	(5)
Part-time Worker	12	(33)	8	(28)	21	(41)	17	(38)
Full-time Worker	80	(40)	87	(34)	66	(47)	74	(44)
Labor Force Attachment	75	(43)	94	(24)	56	(50)	75	(43)
Total Weeks at Home	164	(166)	207	(155)	237	(227)	226	(176)
Weeks at Home Last Year	6	(12)	3	(9)	9	(15)	5	(11)
O*net M Rank	46	(29)	65	(27)	46	(26)	62	(28)
O*net V Rank	43	(30)	65	(26)	48	(26)	66	(25)
O*net S Rank	50	(30)	61	(27)	44	(27)	58	(29)
O*net T Rank	57	(29)	58	(28)	40	(26)	52	(28)
Pre-ASVAB M Rank	43	(26)	76	(20)	43	(25)	75	(21)
Pre-ASVAB V Rank	43	(27)	74	(21)	44	(26)	72	(21)
Pre-ASVAB S Rank	45	(26)	72	(22)	44	(27)	71	(24)
Pre-ASVAB T Rank	48	(28)	67	(23)	44	(27)	67	(25)
ASVAB M Rank	45	(26)	78	(20)	41	(25)	73	(21)
ASVAB V Rank	42	(27)	72	(21)	46	(26)	73	(20)
ASVAB S Rank	49	(28)	77	(21)	40	(24)	65	(23)
ASVAB T Rank	58	(29)	77	(21)	35	(22)	54	(22)
Observations	32,839		9,444		30,720		9,147	
Individuals	2,123		661		2,176		694	

Notes: Standard errors are in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

Source: NLSY. Females and males aged 14-22 in 1979. For detailed definitions see text.

gaps, but also a larger standard deviation, especially when comparing weeks out of the labor force within the last year.

Given the *ASVAB Career Exploration Program* mapping between O*net descriptors and ASVAB test scores, the difference between the occupational skills of men and women can be studied. Original ASVAB test scores and O*net occupational task requirements are converted into percentile ranks within each year using the NLSY cross-sectional weights. Since the NLSY is a representative sample of the US population and

Table 2.2: Gender Gap in Occupational Skill

	Skill Requirements			
Time	Math (1)	Verbal (2)	Science (3)	Technical (4)
Non-College				
1985	2.63***	7.25***	-4.51***	-14.91***
1990	0.34	4.55***	-5.47***	-15.94***
1995	-0.67	3.17***	-6.19***	-17.02***
2000	-2.32***	1.63***	-7.34***	-18.20***
2005	-3.04***	0.70	-7.87***	-19.47***
College				
1990	-0.50	2.22***	-0.70	-4.19***
1995	-2.60***	0.86	-2.59***	-5.41***
2000	-4.10***	-0.57	-4.11***	-6.80***
2005	-4.99***	-0.34	-3.83***	-6.57***

Statistical significance levels: *** p<0.01, ** p<0.05, * p<0.1
Notes: Reporting 5-year averages. For detailed see text.

workforce each survey year this percentile ranking will be consistent.[6] Two measures of ASVAB ranks are reported. Following Cawley et al. (1998) we standardize test scores using two methods: (1) by age alone (labeled "ASVAB"); and (2) by gender and age (labeled "Pre-ASVAB"). The age adjustment is done as all individuals took the test in 1980 and are, therefore, of different age. The gender adjustment is done under the assumption that *ex-ante* men and women are born with the potential to develop the same skill distribution. However, due to economic incentives/preferences/stereotypes men and women choose to specialize in different skills starting at a young age (see Bordalo et al., 2014, and references therein). Specifically, at first glance, women seem to work in higher verbal task occupations and also score higher in verbal tests compared to both men and relative to other skill types.

To broadly summarize the occupation-skill gaps between men and women, Table 2.2 reports the percentile differences across four skill categories by education and gender. This summary converts skills to a percentile rank for each year and then averages over five year intervals by gender. A negative value indicates that women work in occupations requiring less of a given skill then men. Despite convergence along a number of important dimensions (e.g., wages), it appears that the NLSY cohort gender differences actually grew across all skill categories except verbal. These results point

[6] Alternatively, we can also rank individuals according to their test score in 1980 (one-time ranking). However, this does not change the results, as only a biased drop-out from the interview survey would do so. Therefore, assuming the same ranking strategy for individual skills and occupations is our preferred benchmark.

to strong occupation preferences across gender, as the table summarizes occupational skill requirements rather than *ex ante* ability. This last point is especially important given *ex ante* ability is shown to be virtually identical between gender (Goldin et al., 2006). Thus, women with a college degree work in occupations with similar verbal requirements as men, but lower math, science and technical skills. This pattern is repeated for uneducated women, but more skewed toward technical skills.

2.3 Model of Occupational Choice

Beginning with an occupational choice model based on Roy (1951), we model both individuals' skills and their occupational choices. Individuals have n skill types θ^k, $k = 1, , n$, which are drawn from a given distribution at the beginning of their working life. We are agnostic concerning how these skill distributions are initially set, but they may arise from educational choices earlier in life.[7] We account for the potential educational investment in the empirical section. Individuals can choose from a continuum of occupations in period 0 that differ by their skill requirements, Θ_k for all $k = 1, , n$, where $\Theta_k > 0$. That is, all occupations require some skill level of skill type k.

Individuals receive shocks during their lifetime that will force them to temporarily leave the workforce. This is modeled by a simple two-stage Markov process (below) which differs by gender. Individuals are aware of these labor force transition probabilities. The transition probability subscripts denote sequential period status. Specifically, π_{ee} is the conditional probability of being employed today and tomorrow and π_{eh} is the conditional probability of being employed today and staying home tomorrow, where $\pi_{ee} + \pi_{eh} = 1$. As women are more likely to dropout of the labor force (i.e., childbirth) and less likely to return (i.e., child rearing), we set $\pi^m_{ee} > \pi^f_{ee}$ and $\pi^m_{he} > \pi^f_{he}$ to cover these respective observations. Additionally, $\nu^f_e < \nu^m_e$ and $\nu^f_h > \nu^m_e$ match the observed gender-specific employment and, by definition, home probabilities.

$$\Pi^g = \begin{bmatrix} \pi_{ee}\nu^g_e & 1 - \pi_{ee}\nu^g_e \\ 1 - \pi_{hh}\nu^g_h & \pi_{hh}\nu^g_h \end{bmatrix} \text{ for } g = f, m$$

When out of the labor force, skills depreciate by $\delta_{k,h} < 0$. However, when returning to the labor force some skills are recovered $\delta_{k,he} > 0$, but $|\delta_{k,he}| \leq |\delta_{k,h}|$. The atrophy and repair rates are skill-type specific. For most of the theoretical analysis we set learning-by-doing to zero, $\delta_{k,e} = 0$. However, the empirical analysis does account for returns to experience. Assume that $(1+\delta_j) = (1+\delta_{j,he})(1+\delta_{j,h}) < (1+\delta_{i,he})(1+\delta_{i,h}) = (1 + \delta_i)$ for $j > i$. That is, skills are sorted according to their absolute atrophy/repair, $\delta_k \leq 0$, where a higher skill has relatively higher destruction of skill. The heterogeneity

[7] There are many other possible inputs forming these skill distributions.

of skill depreciation suggests that some skills are more insulated against technological innovations.

2.3.1 Agents' Problem

For simplicity, assume a three period model. Individuals work in period 0 for certain and, if they do not drop out of the labor market in period 1, they also work for certain in period 2. Transition probabilities govern if individuals who drops out of work in the middle of their lives will return to work. Individuals draw utility from consumption,

$$U(c) = E\left\{\sum_{t=0}^{2} \beta^t \log(c_t)\right\}. \tag{2.1}$$

There is no savings mechanism and individuals simply consume their income each period, $\log(c_t) = \log(W_t^i) = \omega_t^i$. Individuals that do not work derive utility $b = \log(c_t)$, the value of their home production.[8]

Rather then employ shocks to working or not working, we could have drawn b from a distribution each period, with women having a higher mean then men. However, this will not affect the qualitative results. Therefore, we keep a fixed (gender equal) utility from staying home and allow for differing transition probabilities.

2.3.2 Wages

Assume all possible skill type occupations exist, i.e., there is a continuum of occupations using different skill mixes. The wage an individual receives in the labor market is,

$$\omega_t^i = \sum_{k=1}^{n} \left[\alpha_k \Theta_k^i + \gamma_k \left(\theta_{k,t}^i - \Theta_k^i\right) \Theta_k^i\right]. \tag{2.2}$$

Wages are a function of the returns to each skill type α_k, such that the higher the skill content an occupation requires the higher the wage as long as $\alpha_k > 0$. Agents are potentially penalized if they choose an occupation that requires more skill then they have, $\gamma_k \geq 0$. The interaction between occupation requirement Θ_k^i and skill mismatch $\left(\theta_{k,t}^i - \Theta_k^i\right)$ suggests the penalty (or reward) of mismatch is larger the greater the skill content. This second term will ensure that not all individuals try to match with the highest possible skill content given increasing returns. The second term can also be written as, $\gamma_k \theta_{k,t}^i \Theta_k^i - \gamma_k \left(\Theta_k^i\right)^2$, where the first part is the complementarity between individual skills and occupation requirements, and the second term is the

[8] Alternatively, assume the economy has complete markets so income maximization yields the same results as utility maximization.

general decreasing returns to skill type k. Note that wages without gaps are,

$$\omega_t^i = \sum_{k=1}^{n} \left[\alpha_k \Theta_k^i + \gamma_k \left((1+\delta_{k,e})\theta_{k,t-1}^i - \Theta_k^i\right] \Theta_k^i\right), \tag{2.3}$$

where $\delta_{k,e}$ is the return to experience for skill type k. Wages right after a gap period at $t-1$ are,

$$\omega_t^i = \sum_{k=1}^{n} \left[\alpha_k \Theta_k^i + \gamma_k \left((1+\delta_k)\theta_{k,t-2}^i - \Theta_k^i\right) \Theta_k^i\right]. \tag{2.4}$$

2.3.3 Maximization Problem

The agents maximization problem is to choose the optimal $\{\Theta_k\}_{k=1}^n$ by maximizing expected utility, taking transitions into and out of the labor market into account,

$$\begin{aligned} \max_{\{\Theta_k\}_{k=1}^n} \quad & (1+\beta\pi_{ee}^g(1+\beta)) \sum_{k=1}^{n} \left[\alpha_k \Theta_k^i + \gamma_k \left(\theta_{k,0}^i - \Theta_k^i\right) \Theta_k^i\right] + \\ & \beta^2 \pi_{eh}^g \pi_{he}^g \sum_{k=1}^{n} \left[\alpha_k \Theta_k^i + \gamma_k \left(\theta_{k,2}^i - \Theta_k^i\right) \Theta_k^i\right] + C(b, \Pi^g, \beta). \end{aligned} \tag{2.5}$$

The first term is the net present value of wages if the individual does not have a home spell, the second term is the last period wage if the individual took a gap year in period one, and the last term summarizes the utility from all periods the individual may spend at home.

With a home spell, $\theta_{k,2} = \theta_{k,0}(1+\delta_k)$, the resulting FOCs are (omitting individual superscripts):

$$(\beta_1 + \beta_2)(\alpha_k - 2\gamma_k \Theta_k) + (\beta_1 + \beta_2(1+\delta_k))\gamma_k \theta_{k,0} = 0,$$

where $\beta_1 = (1+\beta\pi_{ee}^g(1+\beta))$ and $\beta_2 = \beta^2\pi_{eh}^g\pi_{he}^g$. Therefore, the optimal occupational choice for skill type k is,

$$\Theta_k^\star = \frac{\alpha_k}{2\gamma_k} + \frac{(\beta_1 + \beta_2(1+\delta_k))}{2(\beta_1+\beta_2)}\theta_{k,0}. \tag{2.6}$$

2.3.4 Comparative Statics

The outcomes of interest in this model are most easily seen from the comparative statics. From Equation (2.6) higher skilled individuals choose more skill-demanding occupations. Moreover, women will sort into lower skill requirement occupations, since $\hat{\beta}^m > \hat{\beta}^f$, where $\hat{\beta} = \frac{(\beta_1+\beta_2(1+\delta_k))}{2(\beta_1+\beta_2)}$. In the extreme case, i.e., men never drop out, $\nu_e^m = 0$, men have much stronger assortative matching then women. Since $\delta_k < 0$ and $\frac{\partial \hat{\beta}}{\partial \delta_k} > 0$. In contrast, women will be increasingly mismatched across skill types with larger absolute depreciation rates. In summary, gap-prone individuals, when maximizing lifetime income, will pick a lower skill occupation when the absolute depreciation rate, δ_k (atrophy plus repair), is larger.

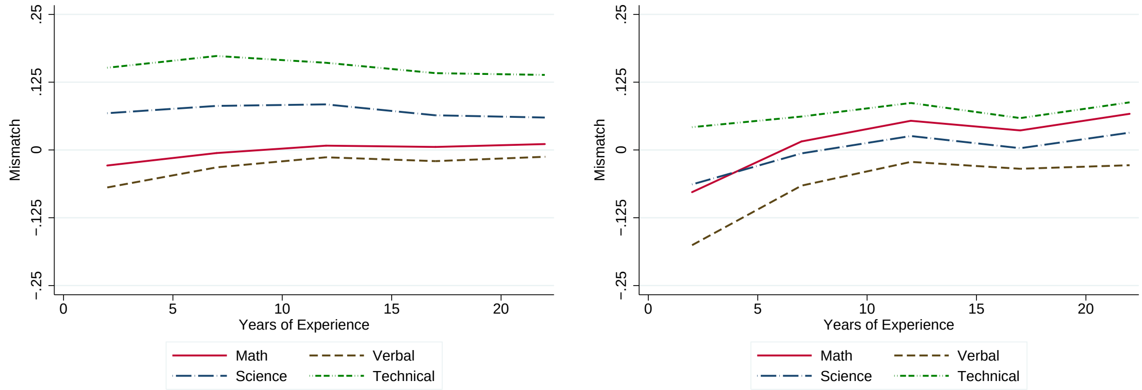
More realistically, assume a finite combination of skill requirements in the economy exist. The set of occupations requires, for example, high math and low verbal or higher verbal and lower math skills. Women would be more likely to sort into the occupations where the dominant skill has lower depreciation rates. For example, if the computer revolution fostered an environment where math and science skills could quickly become obsolete, but there was no similar effect on verbal skills, women would self-select into occupations that require relatively more verbal skills. That is, taking into account the absolute depreciation rates of skills, if women are more likely to take prolonged career gaps, it is optimal for women to choose occupations high in verbal skills and low in math/science skills. Of course, women may also choose different careers due to simple occupational preferences, i.e., women prefer verbal-intensive occupations over technically-intensive occupations. The model presented here only captures the difference in monetary terms, disregarding preference differences.

2.4 Empirical Analysis

2.4.1 Individual Mismatch

Combining the O*net skill content with the ASVAB test scores for the NLSY cohort provides a measure of mismatch in individual skills and occupation requirements. Figure 2.1 graphs the average mismatch between individual skills and occupation requirements by total accumulated work experience. Since we are interested in gender differences, the figure shows the average mismatch of women relative to men in percentiles, where a positive number indicates women are more mismatched than men, and the reverse holds for negative values. The computation is done in five year intervals, with individuals' relative skill ranking, similarly to the occupational rankings, computed for each year. That is, each year we re-rank the individuals remaining in the NLSY sample.[9] The figure assumes that the distributions for males and females are identical in 1980 (the year of the ASVAB test), e.g., the 90th percentile woman has the same skill level as the 90th percentile man. We later relax this assumption, allowing men and women to follow different career paths prior to the exam date.

[9] The results are not sensitive to any of the above ranking assumptions (e.g., sorting individuals according to their ranking in the base year 1979 provides very similar quantitative patterns).



(a) Non-college (b) College

Figure 2.1: Gender Gap Mismatch of Skills by Education

Notes: Average mismatch measures are computed using NLSY 1979 sample weights for all workers (part- and full-time). Results graph average differences between women and men. See text for details.

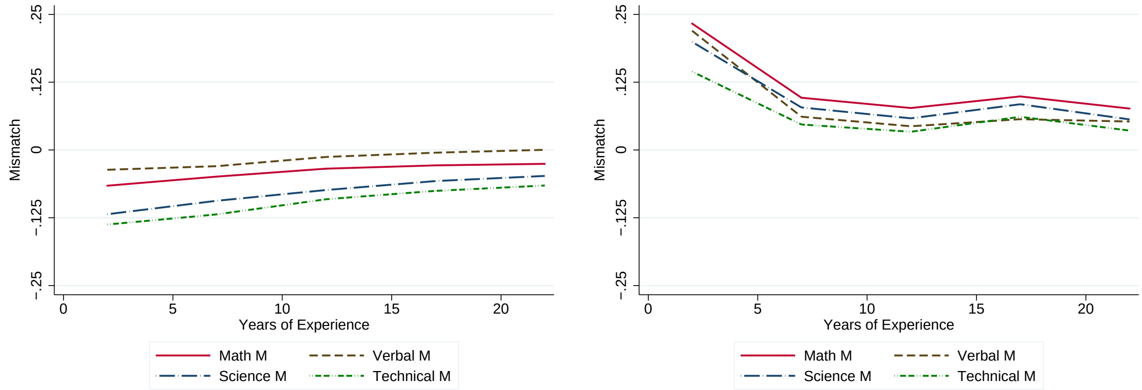
Positive gender gaps exist in math, science and technical fields, with gaps increasing as workers accumulate years of experience. While the math gap for non-college graduates is nearly zero, the science gap for college graduates is approximately zero. In contrast, the opposite gender gap is observed for verbal skills, i.e., women are less mismatched than men.

We expect that individuals will exhibit learning-by-doing and move toward better occupation-skill matches as experience increases. The initial mismatch gap (i.e., workers without work experience) for men follows this theory, with men learning about their skill set and finding more suitable jobs over time (see figure 2.2).[10] This is true for both education groups, although the process is much faster and steeper for college graduates. In direct contrast, women seem to exhibit very little "learning with time" (graph omitted here), explaining the evolution of the gender gap mismatch (see figure 2.1). Part of this higher persistent mismatch for women could be due to an age effect. That is, as women are more likely to take work gaps, groups with low levels of work experience are more heterogeneous across age.

2.4.2 Monetized Mismatch

If skills are underutilized in the labor market, individuals may face lower wages and tenure. Figure 2.1 implies that women could be particularly exposed to any negative effects of skill mismatch across the non-verbal skill dimension. However, observed skill mismatch alone does not necessarily lead to suboptimal outcomes in terms of

[10]Stinebrickner and Stinebrickner (2014) show that students attempt math-heavy college majors and learn about their abilities through failure, moving to more suitable college majors in the process. We postulate that this same mechanism might apply to the labor market.



(a) Non-college (b) College

Figure 2.2: Male Mismatch of Skills by Education

Notes: Average mismatch measures are computed using NLSY 1979 sample weights for all workers (part- and full-time). Results graph average mismatch for men only. See text for details.

maximizing wage returns. One approach is to compare individuals' current match to the optimal skill-matched occupation, and assess the wage difference. To understand which skills are most sensitive to mismatching with respect to wages, Figures 2.4 and 2.5 graph the average "monetized" mismatch. This monetized mismatch concept is partial equilibrium in nature and computes the cost of skill mismatch for women and men assuming no depreciation of skill over time and identical work experience for all individuals. That is, this type of mismatch computation abstracts from individual-specific optimal choices related to skill depreciation rates, work experience accumulation, and any general equilibrium effects affecting skill returns.

Given the economic theory above, combined with data on the occupation skill requirements, individuals' skill rank, hourly wages, and other individual characteristics, we can compute skill prices by running the following regression by year, based on Equation (2.3),

$$\begin{aligned}\log(w_{i,t}) \quad = \quad & \sum_k \alpha_{k,t}\Theta^i_{k,t} + \sum_k \gamma_{k,t}(\theta^i_k - \Theta^i_{k,t})\Theta^i_{k,t} + \sum_k \gamma_{ke,t}\left(\theta^i_k \times \exp^i_t\right)\Theta^i_{k,t} + \\ & \sum_k \gamma_{ke2,t}\left(\theta^i_k \times \left(\exp^i_t\right)^2\right)\Theta^i_{k,t} + X'_{it}\beta_t + \epsilon_{i,t}, \qquad (2.7)\end{aligned}$$

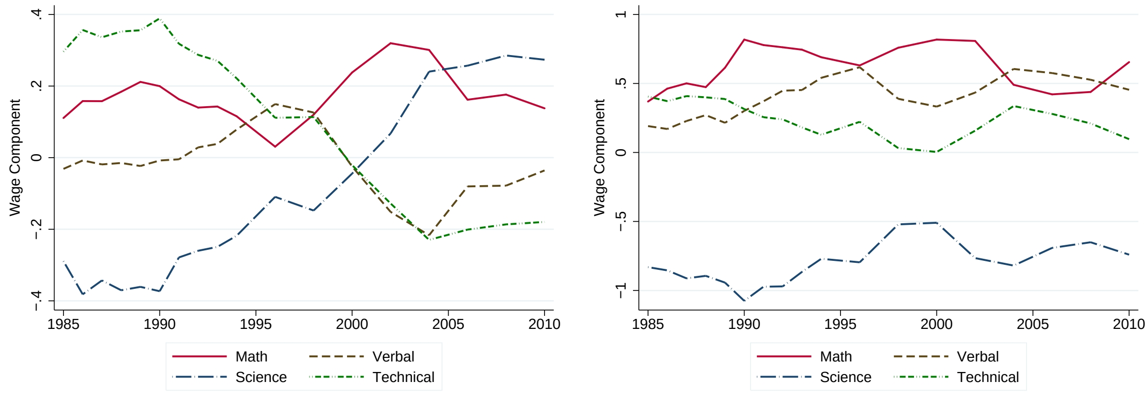
where X_{it} includes age, age squared, race, work experience, work experience squared, marital status, region and degree dummies. $\Theta^i_{k,t}$ is the skill requirement of each occupation from O*net data, θ^i_k is the skill of each individual from ASVAB test scores (we use the percentile rank measure as before) and exp is work experience measured in weeks. Initial skill from the ASVAB test scores interacted with years of experience give current skill levels, $\theta^i_{t,k} = \theta^i_k \times \exp^i_t$. This specification means that α provides the monetized occupational return to math, verbal and science in the economy, and γ provides any wage premium for overqualified individuals or wage penalty for underqualified individuals if $\gamma > 0$. Regressions are run for all years from 1985 to 2010 separately for individuals with and without a college degree. We do this only using full-time working males. Women might be negatively or positively selected into certain occupations, especially if our hypothesis of larger women's mismatch and absolute depreciation differences by skill type is true, potentially biasing any skill prices. In addition, we only include men who have not had substantial working gaps throughout their entire working-lives. We define individuals without substantial working gaps as individuals that have been employed at least 75 percent of their potential working-life (this includes every week since the time of graduation from their highest schooling choice). This sample restriction drops about 18 to 25 percent of the male sample, with 18 percent being dropped in 2006 and 25 percent dropped in 1985. Not surprisingly, when conditioning on educational attainment, this restriction only reduces the college sample by six percent through the whole time period. For further details see Section 2.2 above. We also experiment with more strict definitions, e.g. 80 percent, with results robust to further restrictions. This sample selection ensures the results do not capture the impact of a gap (or the absolute depreciation rate).

Given Equation (2.7), the monetized mismatch is then,

$$\begin{aligned} m_{kt}^{i} &= \sum_{k} \hat{\alpha}_{k,t} \left(\Theta_{k,t}^{\star} - \Theta_{k,t}^{i}\right) - \sum_{k} \hat{\gamma}_{k,t} \left\{ \left(\Theta_{k,t}^{\star}\right)^2 - \left(\Theta_{k,t}^{i}\right)^2 \right\} + \\ & \sum_{k} \left(\hat{\gamma}_{k,t} + \hat{\gamma}_{ke,t}\widehat{\exp}_t + \hat{\gamma}_{ke2,t}\widehat{\exp}_t^2\right) \theta_k^i \left(\Theta_{k,t}^{\star} - \Theta_{k,t}^{i}\right), \end{aligned} \tag{2.8}$$

where $\Theta_{i,t}^{\star}$ is the occupation that would maximize an individual's wage in each year irrespective of any equilibrium effects. This specification accounts for a finite number of occupations, with given math/technical/science/verbal combinations, rather than a continuum of possibilities as presented in the theoretical model. Potential experience in weeks is denoted with $\widehat{\exp}$, which is approximated by the average weeks of experience from the wage regression sample of full-time males without major employment gap history. The monetized mismatch presented here simply assumes a world without depreciation, where men and women have the same work histories in terms of hours.

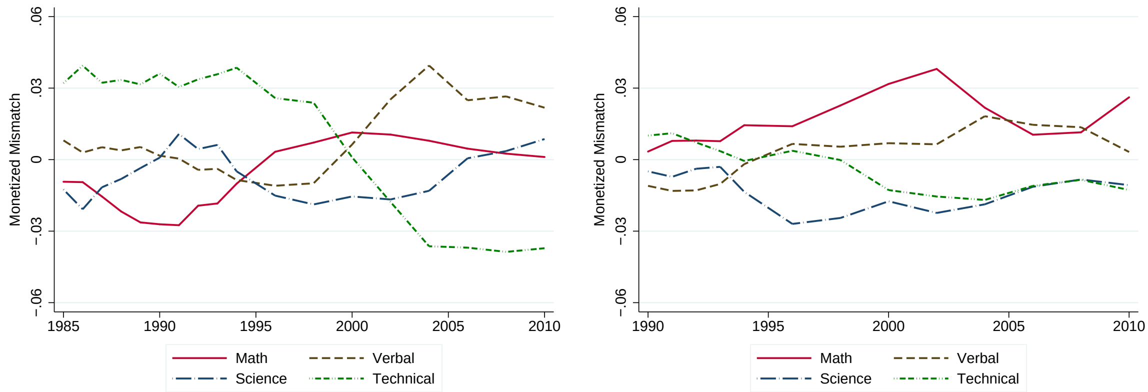
Figure 2.3 shows the average wage component attributable to each skill type for the above wage sample. For college men the skills contribute steady shares to wages, with math and verbal having the largest contribution. Science plays no role in wages, and points to lower average wages for men working in high science occupations. For non-college men, the wage contribution of science has seen the largest upward trend, although technical returns were historically most relevant. Math has always yielded



(a) Non-College (b) College

Figure 2.3: Wage Skill Component

Notes: Skill returns are computed from yearly regressions of hourly log wages of full-time/attached male workers on percentile O*net skill measures, percentile O*net skill measures versus ASVAB test score mismatch, interactions of O*net skill measures with ASVAB test scores and experience/experience squared, experience, experience squared, age, age squared, dummies for race (Black, Asian, and White), region, marital status (married, never married, and other), and last school degree (high school drop out, high school graduate, some college, college graduate, and post-college graduate).



(a) Non-College (b) College

Figure 2.4: The Gender Gap of Monetized Mismatch (Pre-AVSAB)

Notes: Average mismatch measures are computed using NLSY 1979 sample weights for all workers (part- and full-time). Results graph average differences between women and men. See Equation (2.8) and text for details.

positive returns.

Figure 2.4 shows the monetized mismatch from Equation (2.8) for women versus men (i.e., the gender gap) by education level. As before, the figure shows the average mismatch of women relative to men (now) in "monetized" percentiles. A positive number indicates a mismatch of women relative to men that contributes to a positive gender wage gap, with the reverse holding true for negative values.

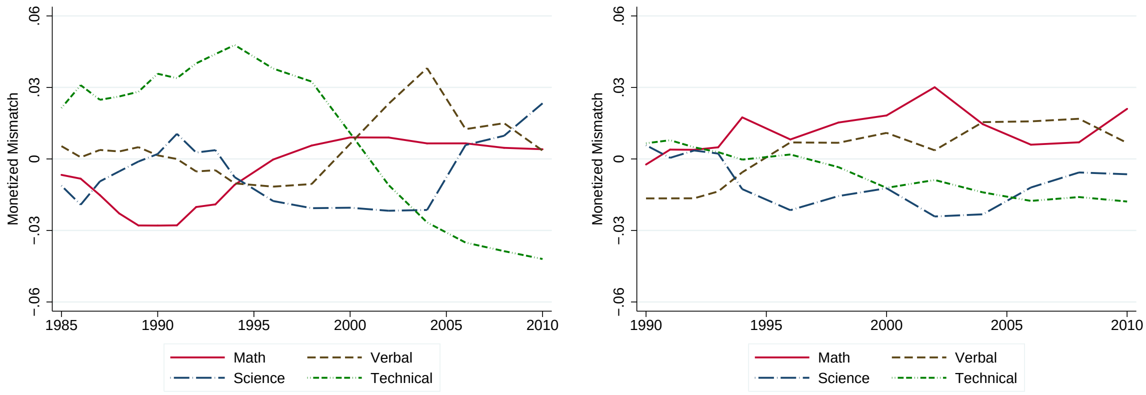
Non-college women saw the largest monetized mismatch in technical skills during the past century. That is, would women have better matched their technical abilities to occupational requirements, *ceteris paribus*, the wage gap of the average uneducated woman would have been roughly three percentage points smaller, although this monetized mismatch disappeared by 2000. Therefore, a decrease in occupations emphasizing technical skills could have potentially contributed to the narrowing gender gap for uneducated women (see also the literature on employment polarization and the disappearing routine occupations, e.g., Autor and Dorn, 2013). For college-educated women, the largest contributor to the gender gap, in terms of skill mismatch, has always been math. Had women been better matched to occupations in terms of their math abilities, the gender wage gap between the average college-educated male and female should have been roughly one to three percentage points smaller. While the monetized math mismatch has been steadily increasing over the sample, the financial crisis corresponds with a temporary dip in the monetized mismatch of math skills.

To explore the idea that women and men may have pursued different skill-specific education, potentially constraining their occupational choices later in life, we specify a regression using standardized test scores adjusted by age only. The age adjustment is necessary as all individuals took the ASVAB test in 1980 and were, therefore, different ages. Not adjusting for gender then allows for the fact that, even by age 16, men and women may already have chosen to emphasize different school subjects leading to different skill outcomes. The reproduced monetized mismatch results (based on Equation (2.8)), using this skill measure, are depicted in Figure 2.5. The skill wage components are virtually the same as the original specification, since this experiment mostly affects the relative position of women to men, and skill prices are only computed using full-time male workers. The qualitative patterns do not change with this post-education skill measure, but the relative gender gap is slightly smaller for college women. This may suggest that women make schooling choices with future career paths (occupations) in mind. However, the differences are not large enough to make any statistical inference.[11]

2.4.3 Atrophy and Repair

The above results beg the question: Why are women mismatched, especially in math skills for college graduates and technical skills for non-college graduates? We investigate if the underlying mechanism points to differences in atrophy and repair rates. To estimate absolute depreciation rates we estimate a wage regression based on Equations (2.3) and (2.4), which is akin to extending the above regression Equation (2.7) to allow for individuals with gaps and the cost of taking a gap. The regression to

[11]It is important to note that these results do not capture any differences in college education choices. Only those schooling choices made before taking the ASVAB test are captured.



(a) Men (b) Gender Gap

Figure 2.5: The Gender Gap of Monetized Mismatch (AVSAB)

Notes: Average mismatch measures are computed using NLSY 1979 sample weights for all workers (part- and full-time). Results graph average differences between women and men. See Equation (2.8) and text for details.

be estimated is then,

$$\begin{aligned}\log(w_{i,t}) \;=\;& \sum_k \alpha_{k,t}\Theta^i_{k,t} + \sum_k \gamma_{k,t}\left(\theta^i_k - \Theta^i_{k,t}\right)\Theta^i_{k,t} + \\ & \sum_k \gamma_{ke,t}\left(\theta^i_k \times \exp^i_t\right)\Theta^i_{k,t} + \sum_k \gamma_{ke2,t}\left(\theta^i_k \times \left(\exp^i_t\right)^2\right)\Theta^i_{k,t} + \\ & \sum_k \gamma_{kg,t}\left(\theta^i_k \times \mathrm{gap}^i_t\right)\Theta^i_{k,t} + \sum_k \gamma_{kg2,t}\left(\theta^i_k \times \left(\mathrm{gap}^i_t\right)^2\right)\Theta^i_{k,t} + \\ & X'_{it}\beta_t + \epsilon_{i,t}, \qquad (2.9)\end{aligned}$$

where, in addition to the previous wage Equation (2.7), gap measures the number of weeks out of the labor force. As in Robst and VanGilder (2000), we use both a cumulative and short-run measure for a gap. The cumulative measure is computed by summing all gaps from the year of graduation, while the current measure only accounts for gaps within the 52 weeks prior to the interview date. Since quadratics on gaps are not statistically significant, the below results do not include the quadratic results. The regressions use the same controls as the computation of skill prices above, e.g., experience, experience squared, age, age squared, dummies for race (Black, Asian, and White), region, marital status (married, never married, and other), and last school degree (high school drop out, high school graduate, some college, college graduate, and post-college graduate), plus year and a part-time dummies if part-time workers are included. An interaction between marital status and gender, since women and men tend to have different "marriage premia," is also included. In line with the literature, ordinary least squares (OLS) estimates from a pooled regression are provided. This

Table 2.3: Full-Time Worker's Depreciation Rates

VARIABLES	LTC (1)	C+ (2)	LTC (3)	C+ (4)	LTC (5)	C+ (6)
Math	-0.054	0.451***	-0.070	0.394***	-0.035	0.475***
	(0.053)	(0.126)	(0.053)	(0.123)	(0.054)	(0.126)
Verbal	-0.004	0.751***	0.002	0.760***	-0.027	0.707***
	(0.047)	(0.108)	(0.047)	(0.105)	(0.048)	(0.108)
Science	-0.365***	-0.992***	-0.359***	-0.937***	-0.367***	-1.006***
	(0.058)	(0.175)	(0.059)	(0.170)	(0.059)	(0.177)
Technical	0.607***	0.351***	0.614***	0.383***	0.595***	0.373***
	(0.041)	(0.127)	(0.041)	(0.126)	(0.042)	(0.130)
Cumm Gap	0.304***	0.481***			0.317***	0.489***
	(0.034)	(0.070)			(0.034)	(0.070)
Last Gap			-0.723***	-1.080***	-0.721***	-1.069***
			(0.079)	(0.264)	(0.079)	(0.264)
Cumm Gap M	-0.075***	-0.095***			-0.072***	-0.087***
	(0.015)	(0.023)			(0.015)	(0.023)
Last Gap M			-0.951*	-2.706**	-0.796	-2.442**
			(0.490)	(1.186)	(0.493)	(1.165)
Cumm Gap V	0.059***	0.047**			0.056***	0.041*
	(0.013)	(0.023)			(0.013)	(0.022)
Last Gap V			0.359	1.958**	0.270	1.873*
			(0.428)	(0.975)	(0.426)	(0.974)
Cumm Gap S	0.043***	0.086***			0.041***	0.082***
	(0.014)	(0.029)			(0.014)	(0.029)
Last Gap S			0.602	0.917	0.478	0.734
			(0.485)	(1.284)	(0.485)	(1.262)
Cumm Gap T	-0.003	-0.009			-0.000	-0.009
	(0.011)	(0.023)			(0.011)	(0.022)
Last Gap T			0.459	-0.370	0.463	-0.408
			(0.342)	(1.176)	(0.344)	(1.183)
Observations	40,411	14,345	40,411	14,345	40,411	14,345
R-squared	0.302	0.319	0.302	0.320	0.305	0.324

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Robust standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

All regressions include experience, experience squared, age, age squared, dummies for years, race (Black, Asian, and White), region, gender, marital status (married, never married, and other), interaction terms between gender and marital status, and last school degree (high school drop out, high school graduate, some college, college graduate, and post-college graduate).

follows given the number of observations and the limited number of gaps observed in the data.[12]

Since the results are estimated using panel data and individuals are follwed over time, there is a potential for serially correlated error terms biasing estimates. Consequently, we also discuss result from generalized least squares (GLS) regressions assuming the error is composed of an unobserved individual effect and a random component, $\epsilon_{i,t} = \mu_i + \nu_{i,t}$.

Table 2.3 includes only full-time workers (results including part-time workers and GLS estimations can be found in Appendix B.3). The general patterns described below are robust to the inclusion of part-time workers or accounting for serially correlated error terms with GLS. The reported variables use the post-ASVAB test scores (i.e., allow for men and women to pre-sort into different study paths). Results with pre-test measures are similar, but usually marginally smaller in magnitude. In addition to gap rates, which are gaps in weeks multiplied by individual skill ranking and occupational skill ranking, the tables also report the return to occupation-specific skills, $\hat{\alpha}$.

Columns (1) and (2) in Table 2.3 show results when only including the cumulative gap measure, columns (3) and (4) shows results for only recent gaps, and columns (5) and (6) show the joint estimates for non-college and college graduates respectively. As in prior research (e.g., England, 1982; Robst and VanGilder, 2000), the cumulative gap measure shows no negative impact on wages; if anything the return to cumulative gaps is positive. However, the cumulative gap measure interacted with math skills reveals some small effects. For example, a college graduate ranked in the 100th math skill percentile and working in the 100th percentile math occupation faces a wage loss of 0.38 percentage points after taking a one month (4 week) gap. In contrast, an individual in the 50 percentile rank in terms of skills and occupation faces a wage penalty of 0.10 percentage points only. The loss for an identical non-college worker would be 0.08 percentage points.

The regression R-squared, at about one-third, is somewhat larger than standard estimates in this literature (see for example Robst and VanGilder, 2000). Given the additional detailed information on skill requirements by occupations and individual skill measures, this is to be expected. Returns to math, verbal and technical skills are, in line with Figure 2.3, positive and statistically significant for college graduates. Only the returns to science have negative coefficients. For non-college graduates only returns to technical skills are large and positive, further corroborating the findings in Section 2.4.2.

As in Robst and VanGilder (2000), a recent gap has a larger wage impact. For example, the college graduate ranked in the 50th math percentile and working in the

[12]Time trends do show similar results, but exhibit somewhat larger standard errors.

Table 2.4: Gender-Specific Depreciation Rates

	Male		Female	
VARIABLES	LTC (1)	C+ (2)	LTC (3)	C+ (4)
Math	0.029	0.884***	-0.004	0.140
	(0.079)	(0.171)	(0.072)	(0.173)
Verbal	-0.310***	0.596***	0.270***	0.862***
	(0.073)	(0.149)	(0.064)	(0.165)
Science	-0.202**	-1.779***	-0.501***	-0.519**
	(0.085)	(0.257)	(0.081)	(0.231)
Technical	0.534***	0.787***	0.479***	0.210
	(0.056)	(0.187)	(0.076)	(0.197)
Cumm Gap	0.272***	0.396***	0.416***	0.577***
	(0.047)	(0.097)	(0.049)	(0.097)
Last Gap	-0.765***	-1.036***	-0.759***	-1.293***
	(0.109)	(0.371)	(0.123)	(0.404)
Cumm Gap M	-0.061**	-0.093***	-0.081***	-0.056*
	(0.024)	(0.031)	(0.019)	(0.033)
Last Gap M	-1.185*	-1.648	0.080	-2.841*
	(0.656)	(1.744)	(0.700)	(1.595)
Cumm Gap V	0.096***	0.076***	-0.003	0.005
	(0.021)	(0.029)	(0.017)	(0.033)
Last Gap V	0.315	0.589	-0.243	3.050**
	(0.607)	(1.516)	(0.613)	(1.353)
Cumm Gap S	-0.020	0.115***	0.096***	0.081**
	(0.021)	(0.044)	(0.019)	(0.037)
Last Gap S	0.788	0.537	0.034	0.892
	(0.611)	(2.158)	(0.795)	(1.480)
Cumm Gap T	0.017	-0.074**	0.006	-0.022
	(0.016)	(0.030)	(0.022)	(0.040)
Last Gap T	0.456	0.093	0.998	-1.460
	(0.435)	(1.785)	(0.885)	(1.692)
Observations	22,663	7,876	17,748	6,469
R-squared	0.255	0.319	0.309	0.281

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Robust standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See Table 2.3 for further details.

50th percentile math occupation faces a wage penalty of 2.71 percentage points when taking a one month gap from the labor force, with the top ranked individual facing a gap four times as large. In general, a college graduate already faces a 4.32 percentage point general penalty for the one month employment gap. While the quantitative results decrease from column (4) to (6), the difference is small. Unlike college-graduates, we do not see a similar math skill-specific penalty gap for non-college graduates. We conjecture that the tasks preformed by a non-college worker with regards to numerical skills have been robust towards technological innovation, while the numerical skills of college graduates have had to adapt to the recent (ICT) technology innovation.

The positive coefficients on verbal skills interacted with gap measures provide a possible explanation for women choosing occupations with considerable verbal skills, if these occupations are immune to skill destruction. However, since selection might be an issue, we also run regressions conditional on gender and education. Table 2.4 shows the gender-specific results (GLS results can be found in Appendix B.3). Columns (1) and (3) report the effects for non-college men and women, and columns (2) and (4) list the effects for male and female college graduates, respectively. Large differences can be easily seen when comparing skill returns across gender within education groups. Given women are more positively matched on verbal skills, the returns to high-verbal occupations are large and positive, while the returns to math are insignificant.

For college-graduates the depreciation rates are very similar to Table 2.3. The verbal gap measure (significant at five percent) completely offsets the math gap measure. One possible interpretation is that women can self-insure against the adverse effects of taking a working gap by picking occupations relatively low in math requirements, but high in verbal requirements. Therefore, these results could explain the observed mismatch patterns from Sections 2.4.1 and 2.4.2 of college graduates.

For non-college women the results show no skill-specific depreciation rates, but instead reveal only general wage loss with gap periods. The skill-specific depreciation rates do not seem to be the main contributor of skill mismatch. An explanation based on stereotypes or preferences could potentially be more relevant (Bordalo et al., 2014).

2.5 Conclusion

We propose and evaluate the idea that women make occupational choices based on skill-specific atrophy and repair with respect to employment expectations. This is a coherent and consistent theory supporting differences between male and female occupational choices. That is, women may choose an occupation with a perceived wage penalty if the penalty for time-off is small. The model presented generates significant economic incentives for women to: (1) strongly prefer occupations that exhibit lower skill-specific depreciation; and (2) pursue the accumulation of skills that are robust to work gaps. The examples provided indicate that the combination of skills within an occupation is more important than the occupation itself. That is, if the largest skill component within an occupation is robust to career gaps, then the other skill requirements' atrophy can be offset.

Using the NLSY panel dataset and O*net occupational skills information, we assess the importance of skill-specific atrophy-repair rates on wages when faced with employment breaks. The model presented leads directly to the empirical exercise and the regression equations employed. The results strongly support the idea that college educated females avoid math-heavy occupations, and pursue verbal-heavy occupations instead. This is due to the high skill atrophy associated with math skills, and the ability of verbal skills to act as "skill insurance" against gaps. Additionally, for college educated individuals, math is the skill most vulnerable to loss during employment gaps, which also implies a slow rebuilding post-break. In contrast, non-college educated individuals experience a much smaller math skill loss. In general, the math content of an occupation appears to be a significant negative for individuals who experience or expect employment gaps, but this is especially true for college educated individuals.

While we find large atrophy-repair rates, the current exercise is unable to estimate how important these rates are for female occupational choices. Moreover, the analysis presented above ignores the general equilibrium effects. That is, if women switch to other occupations, it would change specific skill wage rates. Thus, a general equilibrium model is required to further pursue specific questions, such as: How does skill mismatch contribute to the persistent wage gap? Lastly, we have ignored any educational differences post-ASVAB testing, meaning that education decisions taken in college are not included. In ongoing research we study the educational differences between men and women in college. We take these microeconomic estimates of atrophy and repair by skill type and develop a model to account for equilibrium wages and college education choices. We then ask: How much of the observed gender education differences and the overall gender (wage) gap can be explained by women, accounting for both wage expectations and skill-specific atrophy-repair functions when making educational/occupational choices.

3 Math Matters: Education Choices and Wage Inequality

Joint with Michelle Rendall

3.1 Introduction

> Math is a language of logic. It is a disciplined, organized way of thinking. There is a right answer; there are rules that must be followed. More than any other subject, math is rigor distilled. Mastering the language of logic helps to embed higher-order habits in kids' minds: the ability to reason, for example, to detect patterns and to make informed guesses. Those kinds of skills [have] rising value in a world in which information [is] cheap and messy.[1]

The US has seen a large rise in wage inequality since the mid-1970s (see Figure 3.1). While standard skill-biased technical change (SBTC) can match the average wage trend as measured by the college wage premium (i.e., college/non-college), the large and increasing within-group wage inequality is ignored. In contrast, labor augmenting technical change biased toward math skills can account for a large part of the observed within-group inequality. This paper documents the increasing importance of math skills in the labor market.

Determining which individuals are driving wage inequality and what makes them special yields three facts that support math as a driver of wage inequality:

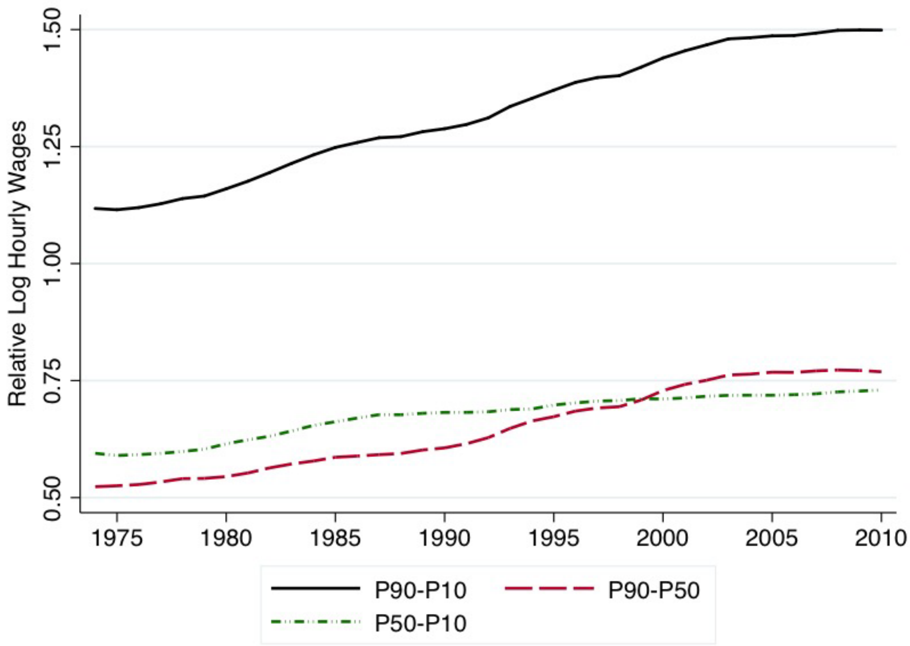
1. Highly quantitative occupations have exhibited increasing relative wages since the mid-1970s.

2. The numerical skill content of occupations and the math content of college majors are highly correlated.

3. Students attempt to study majors with the highest math content, but are constrained by their initial abilities.[2]

[1] Amanda Ripley, *The Smartest Kids in the World: And How They Got That Way*

[2] Note, we use the terms "initial-" and "*ex ante*" ability interchangeably. Initial or *ex ante* ability refers to the ability an individual has upon completing high school.



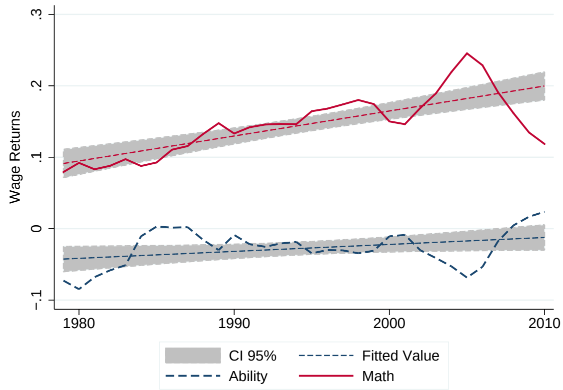
Source: IPUMS-CPS (see King et al., 2010). Log wages are residual wages from a regression of hourly log wages of full-time (at least 35 hours of work and 40 weeks per year) male workers aged 25 to 59 on age, age squared, dummies for education, race, state, and marital status.

Figure 3.1: US Wage Inequality

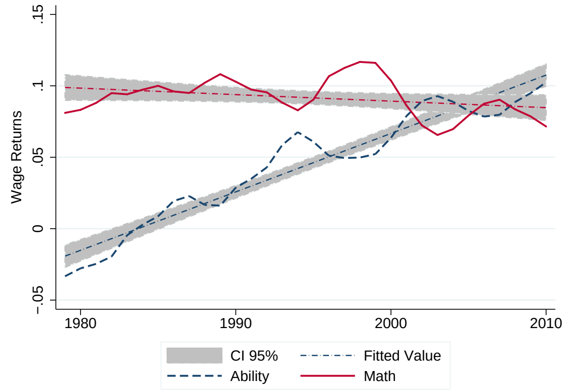
Two distinct trends emerge when measuring the importance of general ability and math skills.[3] The increasing importance of math skills for college educated individuals is shown in panel (a) of Figure 3.2, where increasing wage returns to math skills are set against the stagnant returns of general ability. In contrast, the non-college group has experienced the opposite, with decreasing wage returns to math skills set against increasing returns to general ability. This evidence, combined with the three facts above, suggests that the US economy has not only experienced SBTC, but also math-biased technical change (MBTC), where MBTC cannot be exploited through college attendance alone. More precisely, students who study math-related topics in college will enjoy the largest wage benefits. Thus, the trends presented in Figure 3.2 emphasize MBTC as a mechanism underpinning income inequality between college graduates. MBTC is unlikely to be the main driver of income inequality within the bottom half of the US wage distribution.

The facts and trends discussed above complement the research by Kambourov and

[3]General- and math ability are measured by the Armed Forces Qualification Test (AFQT) from the National Longitudinal Survey of Youth 1979 (NLSY79). General ability is a combination of math- and verbal abilities for the 1979 cohort (see, Altonji et al., 2012a, for details on the construction of standardized AFQT scores).



(a) College Graduates



(b) Less than College

Source: NLSY79. Males aged 14-22 in 1979. Wage returns are computed from yearly regressions of hourly log wages of full-time workers (at least 35 hours in main job) on standardized AFQT test score, standardized AFQT math scores, age, age squared, dummies for race, region, marital status, and whether the individuals lives in a MSA or not.

Figure 3.2: Wage Returns

Manovskii (2009a), who explain a large part of within-group wage inequality by focusing on occupational mobility and the cost of switching occupations. They find that occupational mobility accounts for a significant portion of wage inequality. Similarly, Huggett et al. (2011) study lifetime inequality by decomposing the contribution between initial human capital endowment and "luck," finding that 61.5 percent of the variation in lifetime earnings are due to initial endowments. However, the authors are silent on the decisions (forces) that lead to the differences in human capital at age 23. Thus, we expand on this body of research by modeling the initial conditions that proceed labor force choices. Our approach differs from Kambourov and Manovskii (2009a) and Huggett et al. (2011) by explaining what shapes the individual heterogeneity at the time of occupational choice (i.e., they do not model the formal college human capital accumulation process). We believe the formation of initial conditions (by age 23 when entering the labor market) to be important, as education choices matter for occupation decisions later in life. We show that intra-education group variance is missed when separating the population strictly by educational attainment (college, non-college) alone.

Intra-education group income inequality motivate Altonji et al. (2012b), who find that male electrical engineers earn 51.6 percent more than male education majors, which is comparable to the college wage premium of 57.7 percent. However, due to the empirical focus of their research, they only estimate disaggregated cross-sectional returns to college majors with associated math and verbal SAT scores. The authors note that this area of research is relatively unexplored, but is important for understanding the structural mechanisms underpinning the *ex post* outcomes of higher education. A comprehensive review of the existing empirical studies on the returns to college major can be found in Table 2 of Altonji et al. (2012b). Specifically, the authors note that there is lack of research explaining why individuals choose different education types, and how this translates into occupational choices. A crucial difference between our research and Altonji et al. (2012b) is, by using the information on mathematic skill requirements within occupations from the Dictionary of Occupational Titles[4] (DOT), we show that mathematics-focused majors are highly correlated with *ex post* wage outcomes through the occupational choices available to these majors. We hypothesize that wage inequality is driven, to a large extent, by individuals' initial abilities, which limit education options and, consequently, occupational choices.

Addressing similar wage discrepancies as Altonji et al. (2012b), Silos and Smith (2012) look at the trade-off between acquiring specific and targeted human capital. They concentrate on individuals' choices between education paths leading to specific occupations versus education paths that have broader applicability, and thus more occupational choice. The authors show that policies directed at occupation-specific human capital accumulation lead to lower income growth and lower inequality. MBTC sits within the broader educational transition described above. We emphasize the importance of the skill types accumulated, with particular attention given to mathematics as either a specific necessary ability or as a strong indicator of associated abilities. Those who major in math-intensive areas may initially sort into high-wage occupations, with little incentive of switching (to alternative) occupations.

Carneiro et al. (2011) and Eisenhauer et al. (2013) explore another dimension of intra-education group income inequality and find that the returns to college enrollment are approximately zero for low ability individuals, and possibly negative. MBTC is a potential mechanism that explains this observation, as math-light college majors, without exception, occupy the bottom of the college group wage distribution. Furthermore, there is a significant mass of college graduates with zero and three college math credits.

The idea of SBTC found in Acemoglu (2002), which builds upon the empirical work of Bartel and Lichtenberg (1987) and Autor et al. (1998), formalizes a model in which the labor augmenting technical change is divided along the education dimension (college/non-college). This model has become a workhorse for analyzing and explaining the persistent increase in the relative wages of college graduates. Our work builds on this existing framework by focusing on the distributional wage changes between college graduates. We add a separate mechanism that approximates the specific skills driving wage inequality intra-college graduates.

[4]Dictionary of Occupational Titles, 1977 and 1991

The college attendance mechanism in this paper is loosely based on Hendricks and Schoellman (2014). In that paper the authors look at the discrete education choices of individuals (i.e., high school, some college, and college), focusing on *ex ante* abilities as measured by IQ scores. Their results show that one-third of the college wage premium and one-fourth of its growth is driven by ability ("ability premium"). While looking at ability as a driver of wage outcomes, Hendricks and Schoellman (2014) define broad education categories that mask the sub-group mainly driving wage inequality: the top earning college graduates, who exhibit strong mathematical abilities.

This paper is unique in linking ability, acquired math skills, occupations and rising wage inequality. We aim to explain the evolution of the college graduate wage distribution using a model that emphasizes MBTC combined with the three facts previously discussed. The model revolves around the education choice. Individuals make a choice to attempt college or directly enter the labor market. As we are interested in the outcome of the college education process, we directly assign individuals math credits subject to ability constraints, with some individuals dropping out of college. Math credits characterize each college major in our model. Individuals supply both their *ex post* ability and any acquired math skills to the labor market. Only college graduates can supply the math skills associated with college majors. Firms hire college and non-college labor. Wage inequality is driven by both generic SBTC and specific MBTC.

The theory of MBTC put forth in this paper is complimentary to the task-biased technical change (TBTC) literature first developed by Autor et al. (2003). In order to study a rise in wage inequality in the US, the authors decompose occupation requirements into three task types: manual (hand and finger dexterity), routine (repetitive) and abstract (analytical). Generally, the low, middle and high portions of the income distribution are linked to manual, routine and abstract tasks, respectively. Therefore, a rise in abstract tasks and a fall in routine tasks can generate the wage polarization observed in the US. The abstract measure used in the TBTC literature is composed of two parts, with one capturing the managerial/interpersonal tasks and the other measuring the mathematical skills. MBTC focuses on the second measure, as math skills can be measured at an individual level and cleanly linked to educational choices. Since math skills can be acquired, the supply of these skills can most likely be influenced through education systems and individual choices. In contrast, beyond the significant measurement issues, linking interpersonal skills to acquired knowledge is difficult. Moreover, unlike the TBTC literature, we focus on explaining intra-education group inequality linked to education choices, rather than overall wage polarization in the US. A pronounced wage gap between low- (e.g., communication, psychology, music and drama) and high-math credit majors (e.g., mathematics, engineering, economics, finance) is documented in Table 1 of Altonji et al. (2012b). Given this heterogeneity, low- and high-wage college majors cannot be split along a routine/abstract task dimension.

The focus of this paper is to assess the contribution of MBTC, coupled with innate ability constraints, to the increasing wage inequality in the US from 1980 to 2010. We first calibrate the model to 1980 and then increase skill-biased and math-biased productivity growth rates to match the rise in college attainment and the college wage premium. Given the same innate distribution of skill in 1980 and 2010, we can measure the contribution of MBTC and innate constraints to the rise in wage inequality over time. The calibrated model does will in generating the main trends over time, such as college attendance, college students' abilities and drop out rates. The counterfactuals highlight the strength of MBTC as a determinant of intra-college graduate income inequality. With SBTC alone, the college wage premium is matched through all college graduate wages growing over time, which ignores the observed intra-group trends. In contrast, a combination of SBTC and MBTC matches both the aggregate and intra-group trends observed in the data: the stagnation of wages at the bottom of the college graduate group and the substantial increase in wages for the top college graduates. While high ability individuals respond by studying more math, the bottom of the college graduates are unable to respond because of their limited skill set. Nonetheless, given the rising college premium, more individuals attend college, decreasing average ability (and increasing the variance) of college students. This phenomena feeds back into an even larger math gap between the top and bottom students, increasing wage inequality even further. That is, MBTC is a type of SBTC to which some people cannot respond because of their ability constraint. Given empirical evidence, we argue that math is a valid proxy/measure for these skills.

Wage inequality across different education groups, mathematic requirements of college majors and quantitative occupation requirements form the basis of our model. Thus, Section 3.2 provides a summary of the data facts related to wage inequality, occupations and college majors over time and across cohorts. The general equilibrium model is outlined in Section 3.3, Section 3.4 explains the calibration procedure, and Section 3.5 provides analytical results. Section 3.6 concludes.

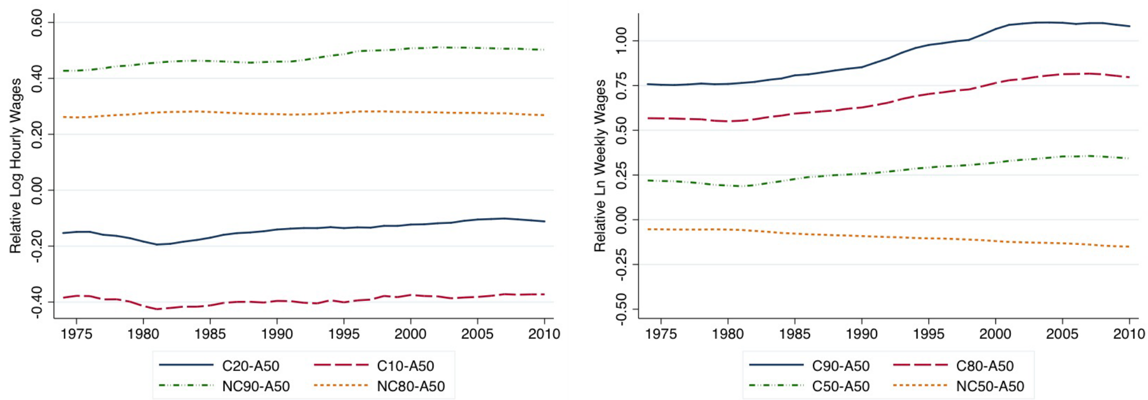
3.2 Data

This research relies on three facts listed in the introduction and expanded upon in this section. Together, these three points present a coherent story of *ex ante* mathematical ability dictating college major options, from which occupations and, ultimately, wages are determined. Those with higher mathematics abilities pursue math-heavy majors and occupations. These particular occupations also enjoy the highest wages.

Furthermore, the math intensive majors that lead to higher wage occupations are increasingly shunned by each subsequent generation of college degree holders. This shift away from math-heavy majors further exacerbates wage inequality.

3.2.1 Who is Driving Wage Inequality?

To illustrate which education-group subsets are driving wage inequality, we use data from the Current Population Survey (CPS), from which the residual of a Mincer wage regression is derived from log hourly wages of full-time, full-year males aged 25-59. The regression controls for age, age-squared, race, marital status, and state of residence (using CPS weights). The unexplained residual for various education-wage groups are compared in Figure 3.3. We use the notationally abbreviations for wage percentiles: “C” for college, “NC” for non-college, and the “A” for the total population (all).



(a) Bottom (b) Top

Source: CPS. See Figure 3.1 for the computation of log hourly wages.

Figure 3.3: Wage Performance

The cross-education wage-group comparisons highlight the importance of high-earning college graduates in driving wage inequality, especially since the mid-1980s. Figure 3.3a compares the residual wages of the C10 and C20 with the NC80 and NC90 wage groups, normalized against the A50 (50th percentile in the total sample). The bottom earning college graduates have significantly lower wages than the upper non-college wage groups and the average wage. All comparisons show a flat or mild divergence. The final comparison within Figure 3.3a shows the bottom college-wage decile has lost ground against the average individual. To put the average into perspective, college-graduates account for about 30 percent of the sample. That is, the average individual (A50) is a non-college graduate, just above the average non-college (NC50) individual (see Figure 3.3b bottom line).

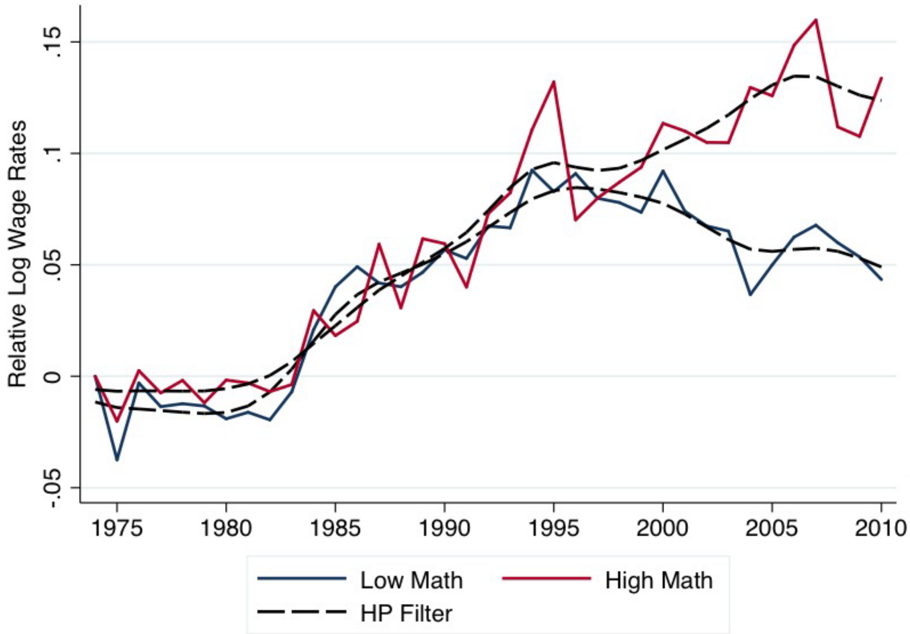
In contrast to Figure 3.3a, Figure 3.3b compares the residual wages of the middle and top college, and the middle non-college wage groups with the average individual. The C90 wage group has outpaced the C50 wage group by 20 percent since the mid-1970s. This is a remarkable performance considering the C50 wage group increased their wage premium against the median individual by approximately 12 percent. For reference, the college wage premium grew by 22 percent. The implications of this figure are summarized in two points: (1) the average college graduate is outpacing all other groups, but (2) the top college graduates are sprinting ahead of everyone. Thus, a large part of wage inequality growth is driven by the top college-wage groups, while the bottom college-graduates are left behind compared to a large share of non-college graduates.

3.2.2 Fact 1: MBTC

As within-group wage inequality is driven by the top earners, it is important to pin down the characteristics that defines this group. The idea of MBTC is compelling when considering the relative returns to math and general ability previously presented in Figure 3.2 for the National Longitudinal Survey of Youth 1979 (NLSY79) cohort. For college graduates, labor market returns to math have increased over time, with the evidence pointing to MBTC as the driver. However, given the survey design of the NLSY79, it is impossible to disentangle time effects from life-cycle effects. In addition, estimating MBTC together with SBTC requires a time series dataset of the US economy. As the American Community Survey (ACS) only has cross sections for 2009 and 2010, we use the DOT numerical requirements to exploit the time series dimension of the CPS to further understand the importance of MBTC.[5] Figure 3.4 shows relative log wage rates of college graduates split equally between high- and low-math occupations[6] relative to non-college wage rates. The wage rates are computed using efficiency units of labor, with more detail on the precise computation found in Appendix C.2. The relative wages of high-math occupations began to diverge in the mid-1980s, which is consistent with the beginning of large-scale personal computer adoption, a main driver of SBTC (Autor et al., 1998). Results suggest that, for college graduates, labor augmenting technical change on high-math occupations has grown about 16 percent per annum faster than on low-math occupations. Appendix C.2 provides details on the estimation method for both SBTC and MBTC over time using the CPS.

[5] Appendix C.1 provides additional detail about the DOT aptitude measures used in this paper.

[6] The 50% split means that the cutoff between low- and high-math occupations is such that 50% of college graduates in 1974 work in high-math occupations. However, the results are not sensitive to this cutoff, e.g., using a top-third versus bottom two-thirds split yields similar quantitative results.



Source: CPS. See Appendix C.2 on the detailed computation of log wage rates.

Figure 3.4: Relative Log Hourly Wages of High- to Low-Math Occupations

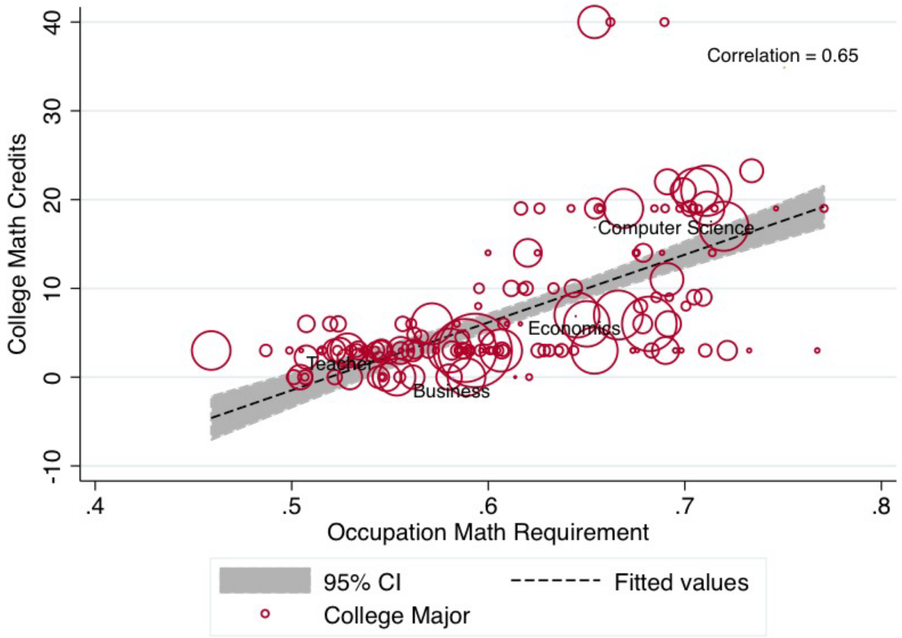
3.2.3 Fact 2: College and Work

Figure 3.5 depicts the relationship between college majors' math credits and the numerical skill requirements of occupations in 2010 for individuals aged 23 and 62. The figure uses the individual-level observations with college major and occupation information from the ACS, combined with the DOT numerical job requirements.[7] All individuals are first grouped by their college major and the average occupation math requirement is computed, as there are multiple occupation outcomes within each college major. This figure shows that occupation-specific math skills are highly correlated with college-level math credits by college major, with a 0.65 correlation coefficient.

3.2.4 Fact 3: College Math and Ability

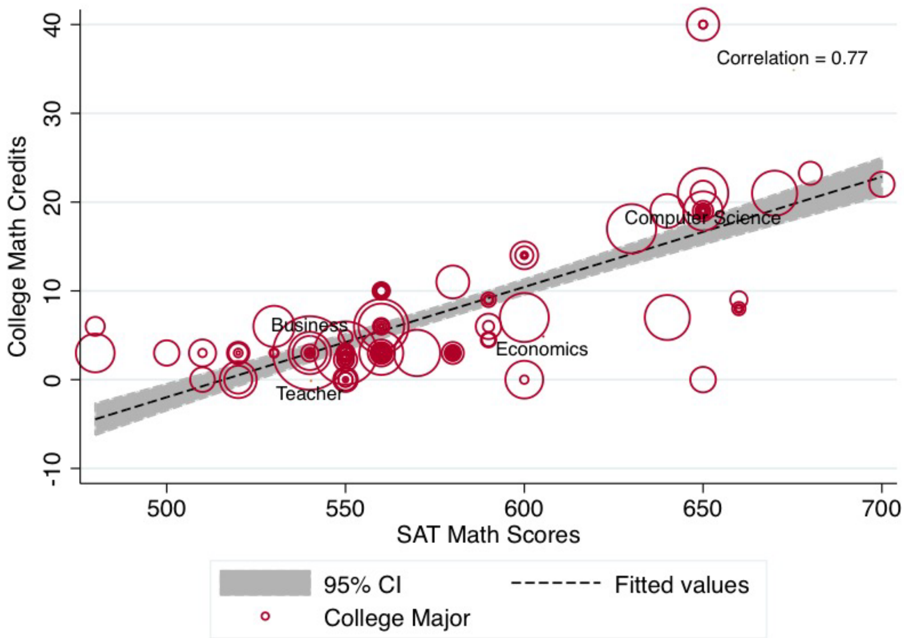
Looking at the initial characteristics that lead to college major sorting, Figure 3.6 merges college-level math credits and mean SAT Math scores by college major from the National Center for Education Statistics (NCES) to individuals in the ACS. The results illustrate that *ex ante* abilities are correlated with college-level math credits with a correlation coefficient of 0.77. Thus, those with high-math abilities prior to college, as measured by the average SAT Math scores of those graduating within a

[7]The ACS 2010 is used throughout. The ACS 2009 is the first year in which college major is included. Note that the trends observed in the ACS 2010 are virtually identical to the ACS 2009.



Source: ACS, NCES, DOT. Full-time, full-year, males, age 23-62.

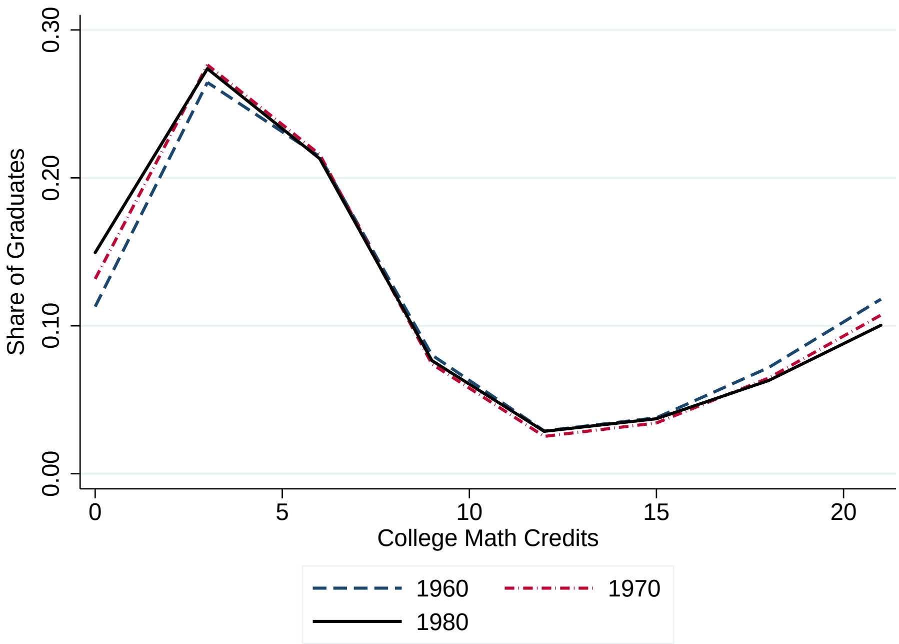
Figure 3.5: Occupation Math Requirements and College-Level Math Credits by Major



Source: ACS, NCES. Full-time, full-year, males, age 23-62.

Figure 3.6: SAT Math Scores and College-Level Math Credits by Major

specific college major, are more likely to graduate from math intensive college majors, as measured by college-level math credits.



Source: ACS, NCES. College graduate males by birth cohort.

Figure 3.7: College Major Graduation Share by Cohorts

3.2.5 Other Measures

The ACS offers a cross-sectional snapshot of college majors in 2010, from which we construct a measure of how individuals' college major choice has evolved. This assumes that most individuals do not go back to school beyond the age of 30, and that the ACS sample is representative of the population at every age group. Figure 3.7 illustrates the changes in degrees obtained, as measured by the share of graduates at each math credit level of three sample cohorts between the 1960s and 1980s. The figure shows a general and persistent pattern of college graduates shifting away from relative high-math majors to low-math majors since the 1960s. Given the general trend toward MBTC in the labor market, this pattern may seem puzzling. However, the leftward shift in the quality of college students, suggested by Heckman and Mosso (2014), can also explain a shift towards the left in Figure 3.7. The hypothesis to be tested in this paper is to precisely determine the importance of MBTC combined with this shift towards low-math majors in generating the observed increase in wage inequality within college graduates.

Table 3.1 shows how other measures of ability are correlated with log wages, college math credits and the usual SAT measures of *ex ante* math and verbal ability. AbilityG, AbilityV and AbilityN are the DOT measures for general, verbal and numerical aptitudes, respectively.[8] The results presented here are for all individuals, with similar

[8]The DOT measures of ability are detailed in Appendix C.1.

Table 3.1: Ability Measures and Wages

	log(w) (1)	AbilityG (2)	AbilityV (3)	AbilityN (4)	SATM (5)
AbilityG	0.601***	1			
AbilityV	0.563***	0.966***	1		
AbilityN	0.846***	0.766***	0.719***	1	
SATM	0.609***	0.598***	0.571***	0.725***	1
SATV	0.387***	0.565***	0.551***	0.502***	0.796***

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: ACS, NCES, DOT. Full-time, full-year, males, age 23-62.

results across cohorts. However, the correlation between log wages, general- and verbal ability are smaller for younger cohorts. For example, individuals aged 28 to 32 in 2010 (the 1980 cohort) have a correlation between wages and general ability of 0.46, verbal ability of 0.43 and math ability of 0.78. The population correlations for these same measures is 0.60, 0.56, and 0.85, respectively (see Table 3.1). This difference may be due to an age-effect when first entering the labor market, i.e., individuals learn about different occupations and their skill requirements through experience.

The correlation for math measures (SATM, AbilityN) are 40-60 percent greater than non-math measures. While this is possibly due to noise within the non-math measures, the correlation between SATM and SATV is 0.80. This high correlation between math and verbal scores, despite the much higher correlation between log wages and math scores, further highlights the special significance of math as either a direct or indirect measure of high-return skills in the labor market. Note that, with respect to the timing of college-level education, the DOT ability measures are *ex post* assessments, whereas the SAT ability measures are *ex ante* assessments.

3.3 Model

As this research is focused on the evolution of wage inequality over time, a general equilibrium model is required to provide the time dynamics that would invariably be ignored by estimating a regression model on the cross sections of available data. The overlapping generations model used here features a unit mass of finitely lived agents and a single representative firm.

3.3.1 Individuals

The model is loosely based on Hendricks and Schoellman (2014). However, the schooling decision is modified to incorporate a choice between "college majors" and

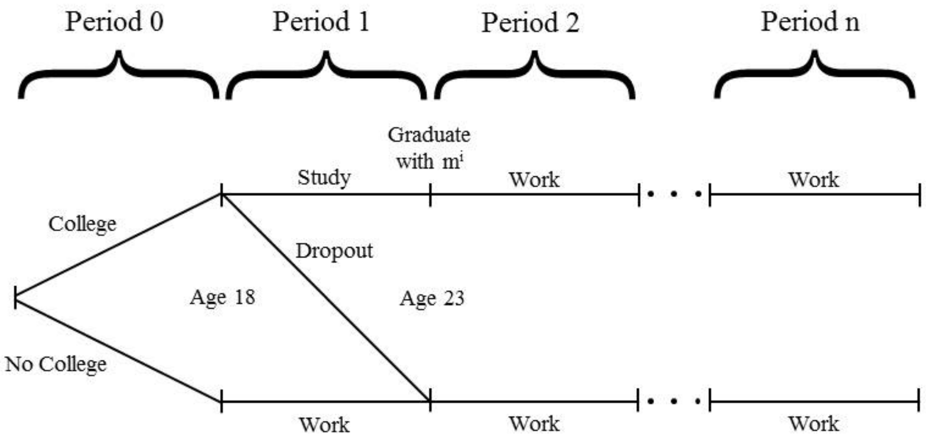


Figure 3.8: Timeline for Individuals

the fact that individuals face constraints based on their ability. Figure 3.8 provides the basic model timeline for individuals. For each generation, there are two primary periods with all decisions taking place in period 0 and the realization of the decisions occurring in period 1. During period 0, individuals choose to either enter or forgo college (college or non-college). At the beginning of period 1, individuals are about 18 years old. If they attempt college there are two outcomes: (1) drop out/fail or (2) graduate with a specific degree characterized by specific acquired math skills, m^i. Both college graduates and dropouts enter the labor force in period 2, with individuals who drop out/fail losing one period of income as an opportunity cost of attempting college. Each subsequent period, individuals supply their general ability and acquired math skills to firms in exchange for wages.

Individuals' decisions within the model are straightforward. Individuals decide to pursue college education or enter the labor market in period 0. This is the only choice available to individuals in this model. Dropping out is determined by a single ability cutoff. The college major "choice" is simplified to a direct mapping from human capital, h_m, to acquired math skills, m, without an explicit choice. Arcidiacono et al. (2012) and Zafar (2013) find strong evidence that men optimize their education choices in order to earn the highest wage return. Also exploring men's education choices, Paglin and Rufolo (1990) find empirical evidence that men choose college majors based on their math ability. This result is similar to that of Stinebrickner and Stinebrickner (2014), who show that many people attempt math-heavy college majors, but learn about their abilities through failure. These people move into college majors with lighter math loads, with the failure and "drop" process repeating until the student's ability is matched to the math content of the college major or they drop out. Thus, for men, combining these results points to a direct mapping from initial ability to the highest possible college math outcome.

The education choice is determined by weighing the financial benefits against the utility and opportunity costs (lost wages) of studying. Markets are complete, such that income maximization and consumption maximization yield the same results, with discounting of $\beta = \frac{1}{1+r}$. Thus, the individuals' objective function is:

$$\max_{s^i} \quad \left\{ \sum_{t=2}^{N} \left(\frac{1}{1+r} \right)^t E\big[p(\theta)\omega_{et}(\theta, \theta_m) + (1 - p(\theta))\omega_{ut}(\theta)\big] - \zeta^i, \right. \\ \left. \sum_{t=1}^{N} \left(\frac{1}{1+r} \right)^t E(\omega_{ut}(\theta)) \right\}. \tag{3.1}$$

Individuals make a schooling choice, $s^i \in \{e, u\}$, between attending college or not. Individual wages are given by ω^i_{jt}, with $j = \{e, u\}$ denoting the college graduate (educated) or non-college (uneducated) outcomes. Wages are a function of individuals' initial general ability, θ^i. College-graduate wages are, in addition, also a function of individuals' initial math ability, θ^i_m. The probability of graduating from college is represented by p and depends on an individuals' initial general ability, θ^i. Individuals are heterogeneous across initial general ability, θ^i, initial math ability, θ^i_m, and taste for college, ζ^i.

While initial ability is defined in terms of θ^i and θ^i_m, agents are aware that ability translates into human capital, h^i, through a noisy process, that affects their performance both at school and at work. This process is defined as $h^i = \exp(\theta^i + \epsilon^i)$ and $h^i_m = \exp(\theta^i_m + \epsilon^i)$ for general and math human capital, respectively. Individuals' *ex ante* estimate of their general and math human capital is given by $\hat{h}$ and $\hat{h}_m$. Students generally overestimate their human capital when making schooling choices, which we call overconfidence (for an empirical motivation see Bordalo et al., 2014, and references therein). This overconfidence is seen in individuals' *ex ante* general and math human capital estimates: $\hat{h} = \exp(\theta + \hat{\epsilon})$ and $\hat{h}_m = \exp(\theta_m + \hat{\epsilon})$, where $E(\hat{\epsilon}) > E(\epsilon)$.

College is not reversible and dropping out occurs only if an individual does not meet the minimum graduation requirement set by $\overline{h}$. I.e., $p(\theta^i) = 1$ if $h^i(\theta) > \overline{h}$ otherwise $p = 0$ and the individual drops out of college.

Math human capital has no value in the labor market unless an individual studies math in college. We will approximate this college major matching process by allocating math credits directly to individuals based on their ability. Acquired math skill in college, $m^i(h_m)$, is an increasing function of math human capital, $\frac{\partial m^i(h_m)}{\partial h_m} > 0$.

The wages for college educated individuals are determined by,

$$\omega^i_{et} = w_{et} \left(w_{het} h^i + w_{met} m^i(h_m) \right) \exp(\eta^i_t), \tag{3.2}$$

where w_{et} are general wage returns to a college degree for general human capital (indexed h) and acquired math skill (indexed m). A transitory luck component, η, is drawn each period.

The wages for uneducated individuals are,

$$\omega_{ut}^{i} = w_{hut} h^{i} \exp(\eta_{t}^{i}), \tag{3.3}$$

as math human capital has no value unless refined in college. Uneducated individuals face the same transitory luck component, η, as educated individuals.

3.3.2 Firms

A representative firm hires college and non-college labor to produce a final good (Y_t). The production function is a CES between non-college and college labor. College labor is a nested-CES between general human capital and math. Non-college labor, by definition, only supplies general human capital.

$$Y_t = \left[\alpha L_{hut}^{\nu} + (1-\alpha)\left[\lambda \left(A_t L_{het}\right)^{\rho} + (1-\lambda)\left(A_t M_t L_{met}\right)^{\rho}\right]^{\nu/\rho}\right]^{1/\nu} \tag{3.4}$$

The elasticity of substitution between education types is $\frac{1}{1-\nu}$. The elasticity of substitution between general human capital and acquired math skills is $\frac{1}{1-\rho}$. Labor shares are comprised of two components for educated individuals, general human capital and acquired math skills, and general human capital alone for uneducated individuals. Formally, labor shares are defined as,

$$L_{hjt} = \int_i (\mathbf{1}_{(s^i=j)} h^i) di, \ \forall j = e, u \quad \text{and} \quad L_{met} = \int_i (\mathbf{1}_{(s^i=e)} m^i) di.$$

A_t is skill-biased technical change (SBTC) and M_t is math-biased technical change (MBTC) over time, with growth rates γ_{at} and γ_{mt} for SBTC and MBTC, respectively. Thus, SBTC is $A_t = (1+\gamma_{at})A_{t-1}$ and MBTC is $M_t = (1+\gamma_{mt})M_{t-1}$.

The model solutions follow from the firm's cost minimization problem. If we define college labor output as,

$$Y_{et} = \left[\lambda \left(L_{het}\right)^{\rho} + (1-\lambda)\left(M_t L_{met}\right)^{\rho}\right]^{1/\rho},$$

with a price of $p_{et} = w_{et}$, given perfect competition, then the relative demand for college labor output from the firm's minimization is,

$$\left(\frac{Y_{et}}{L_{hut}}\right)_{demand} = \left(\frac{A_t^{\nu}(1-\alpha)}{\alpha} \frac{w_{hut}}{w_{et}}\right)^{\frac{1}{1-\nu}}. \tag{3.5}$$

Firms demand relatively more college labor when the wage rate decreases or college labor productivity (A_t) increases. The relative demand for acquired math skills ("college math") from the firm's solution is,

$$\left(\frac{L_{met}}{L_{het}}\right)_{demand} = \left(\frac{M_t^{\rho}(1-\lambda)}{\lambda}\frac{w_{het}}{w_{met}}\right)^{\frac{1}{1-\rho}}. \tag{3.6}$$

Firms demand relatively more college math when the wage rate decreases or the productivity of acquired math skills (M_t) increases.

3.3.3 Equilibrium

The general equilibrium conditions are dependent on the individuals' and the firm's optimization problems.

An equilibrium, given wage rates $\{w_{hut}, w_{et}, w_{het}, w_{met}\}$, is defined by:

1. The education choice, $s^i = \{e, u\}$, that maximizes the individual problem, subject to the graduation constraint $h^i \geq \overline{h}$;
2. The demand for labor $\{L_{hut}, L_{het}, L_{met}\}$, that minimizes the firm's production cost; and
3. Labor markets clear, both for general human capital, $(L_{hjt})_{demand} = (L_{hjt})_{supply}$ for $j = \{e, u\}$, and for college math, $(L_{met})_{demand} = (L_{met})_{supply}$.

3.3.4 Dynamics

To compute actual wage rates, the final good price is normalized to one, $p_t = 1$. Using the unit cost of producing one unit of output, it is straight forward to derive all wages from Equations (3.4)-(3.6). The wage rates of college labor input are,

$$w_{het} = w_{et}\lambda^{1/\rho}\left(1 + \frac{M_t^{\rho}(1-\lambda)}{\lambda}\left(\frac{L_{met}}{L_{het}}\right)^{\rho}\right)^{(1-\rho)/\rho} \tag{3.7}$$

and

$$w_{met} = w_{het}\frac{M_t^{\rho}(1-\lambda)}{\lambda}\left(\frac{L_{het}}{L_{met}}\right)^{1-\rho}, \tag{3.8}$$

where

$$w_{et} = A_t(1-\alpha)^{1/\nu}\left(1 + \frac{\alpha}{A_t^{\nu}(1-\alpha)}\left(\frac{L_{hut}}{Y_{et}}\right)^{\nu}\right)^{(1-\nu)/\nu}. \tag{3.9}$$

SBTC (A_t) increases the returns to all college labor. There are two channels that increase the returns to math college, (1) a direct technical change effect, and (2) an indirect supply effect. First, MBTC (M_t) increases the returns to college math directly.

Second, a relatively faster increase in general human capital compared to the supply of college math, $\frac{L_{het}}{L_{met}}$, given an elasticity parameter of $\nu < 1$, also increases the wage rate on college math, w_{met}. Therefore, consistent with our main hypothesis, if individuals are constrained in learning math, or new college entrants are unable to learn/study math, it is possible that the returns to math increase faster than the returns to college, leading to a larger spread between the top and bottom percentile wages within the college educated group. As a consequence, the larger the absolute number of college entrants, given that every new marginal entrant will have a lower ability level, the larger the post-education wage inequality. For completeness the wage rate of uneducated workers is determined by,

$$w_{hut} = w_{et} \frac{\alpha}{A_t^{\nu}(1-\alpha)} \left(\frac{Y_{et}}{L_{hut}} \right)^{1-\nu}, \tag{3.10}$$

where the non-college relative wage decreases with SBTC, which is consistent with the SBTC literature.

3.4 Calibration

We calibrate the model to 1980. The model parameters can be grouped into four categories: (1) standard parameter values, $\{\beta, N\}$; (2) individual-specific parameters, $\{\mu_\theta, \sigma_\theta, \sigma_\zeta, \hat{\mu}_\epsilon, \sigma_\epsilon, \sigma_\eta\}$; (3) college-specific parameters, $\{\delta_0, \delta_1, \overline{h}, \overline{m}\}$; and (4) firm-specific parameters $\{\alpha, \lambda, \nu, \rho\}$. Each category of parameters is discussed in separate subsections below. The calibration procedure estimates all parameter values jointly. To generate the time trends, the growth rates of productivity $\{\gamma_{jt}\}$ (SBTC and MBTC) are calibrated to match the rise in college attainment and the college wage premium.

3.4.1 General Parameters

The model has five-year time periods and is simulated for seven periods, from 1980 to 2010. We start the model economy at 1980 for two reasons: (1) the 1960 cohort of the NLSY79 is the first reasonable target available, i.e., individuals making their college decisions in 1980; and (2) the Vietnam War distorted the college decision of cohorts born before 1960, as the draft could be avoided by college enrollment (see Lemieux and Card, 2001). In addition, the decompositions in Section 3.2 show that income divergence began around 1980, with a relatively flat trend between 1975 and 1980. The model uses a standard discount factor of $\beta = 0.9$ per period, which implies a discount rate of approximately two percent per year.

We set $N = 9$, meaning that individuals live for nine periods after the college/no-college decision. Thus, each period contains nine generations and the modeled working life-time of an individual covers the equivalent of 45-years. As the period 0 decision is assumed to take place around the age of 18, the model covers the age range of 18 to 63. The simulation accounts for the baby boom/bust that generates different cohort sizes. However, the results are not sensitive to these cohort size differences.

3.4.2 Individual and Education Specific Parameters

The individual and education parameters interact directly. Therefore, this subsection discusses these two parameter groups together. Given that the NLSY79 was administered to individuals aged 14 to 22 in 1979, and model simulations start in 1980, we drop the youngest individuals for NLSY79 targets described below. That is, we match the 1960 cohort definition, only including individuals born before 1963.

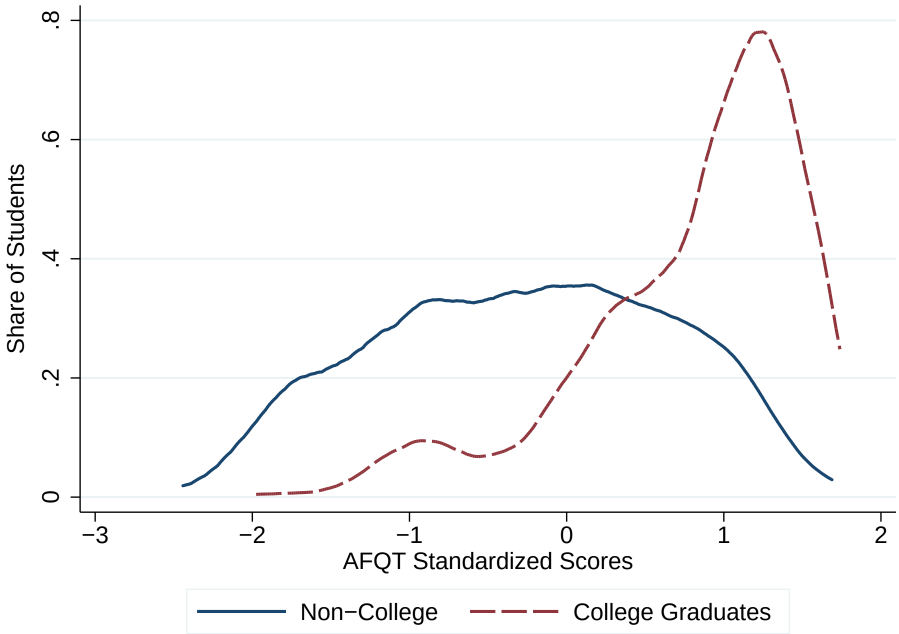
Initial ability and human capital. The two types of initial ability, general (θ) and math (θ_m), are distributed normally. Formally, $\theta \sim N(0, \sigma_\theta^2)$ and $\theta_m \sim N(0, \sigma_\theta^2)$. The correlation ($\rho$) between initial general ability and initial math ability is set to $\phi = 0.9367$, matching the correlation between SAT I and SAT I Math scores in the ACS, $E(\theta_m) = \phi\theta$.

To generate the mapping between initial ability measures (θ and θ_m) and human capital (h and h_m), the noisy process ϵ is assumed $N(0, \sigma_\epsilon^2)$. The *ex ante* process $\hat{\epsilon}$ is also assumed $N(\hat{\mu}_\epsilon, \sigma_\epsilon^2)$.

Schooling choice. Preferences for studying impact the initial college/no college schooling choice. Study preferences are defined by ζ, which can be considered an individual's taste for college. While individuals sort into college based on their initial ability, Figure 3.9 shows that this sorting is not perfect. Thus, $\zeta \sim N(0, \sigma_\zeta^2)$, where a negative ζ is a cost and a positive ζ is "love" for studying.

Schooling outcome. There are two outcomes for those who attempt college education: (1) dropout/failure or (2) graduate in a college major characterized by math credits. In the model, the dropout rate is governed by a minimum human capital standard, $\overline{h}$, which is assumed constant over time. Those who do not drop out of college accumulate a representative measure of acquired math skill in college (m^i), with functional form,

$$m^i = \min\left\{\overline{m}, \max\left(0, \delta_0 + \delta_1 \exp(h_m)\right)\right\}. \tag{3.11}$$



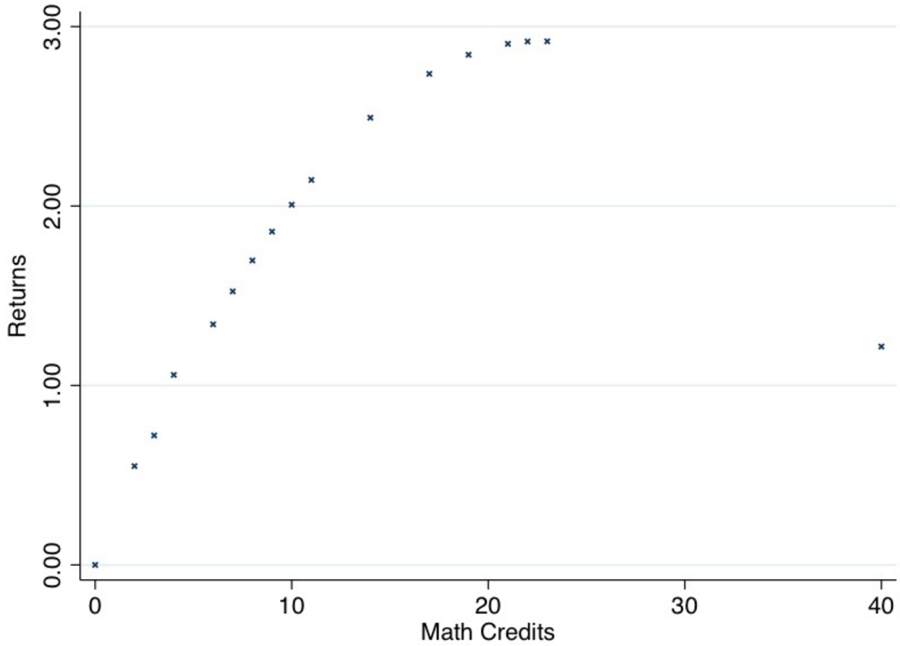
Source: NLSY79. Males born before 1963 (1960 cohort). Standardized test scores as computed by the method of Altonji et al. (2012a).

Figure 3.9: Ability Distribution by Education Type

Math acquired in college is subject to a cap, $\overline{m}$, which is set to match the share of individuals with 21 or more math credits in the 1960 cohort. The choice of 21 math credits is driven by returns to math credits in the ACS sample. A regression of math credits and math credits squared on wages, controlling for a number of characteristics, for full-time full-year male employees, suggests an increasing and concave relationship for the returns up-to 21 college math credits (see Figure 3.10). The returns to additional credits above 21 flattens and then drops sharply at 40 credits. These 40-credit college majors are primarily math majors. We argue that numerical skills and the tools learned in mathematics are valuable on the job market, but we do not, *per se*, think that esoteric math courses (e.g., chaos and dynamical systems) are the main driving force. However, the US share of college graduates with 40 credits is small.[9]

The number of college graduates studying zero math credits can be seen in Figure 3.7. These individuals will benefit from SBTC, but not from MBTC. We define δ_0 such that the share of college graduates from the 1960 cohort with zero math credits is matched. Given the mapping function for m^i, δ_0 will be a negative value.

[9]Only 1.6 percent of college graduates within the entire 2009/2010 sample obtain 40 math credits.



Source: ACS, NCES. Math credit returns are computed from a regression of hourly log wages of full-time (at least 35 hours of work and 40 weeks per year) male workers aged 25 to 59 on math credits, math credits squared, age, age squared, dummies for education, race, marital status, and year.

Figure 3.10: ACS Returns to College Math Credits

Additional wage component. The model features a standard luck component (e.g., Storesletten et al., 2001) in wages over the life-cycle. For the precise process used in this paper, see Guvenen and Kuruscu (2010). This luck component is i.i.d. with mean zero and standard deviation σ_η.

Remaining moment conditions. The six structural parameters that do not have a one-to-one mapping to empirical moments are estimated by matching seven US data targets simultaneously. The seven moments that govern individual actions pertain to the 1960 cohort (i.e., individuals making education choices in 1980) or the year 1980. We group the moments that we believe to be particularly informative of a given parameters.

- $\overline{h}$ is governed by the college dropout rate in 1980. Although there is a wide range of estimates for the dropout rate, we use estimates by Bound et al. (2010) for two reasons: (1) the authors provide estimates disaggregated by gender; and (2) their definition aligns with our interpretation of college dropouts. That is, the college dropout rate defined as the share of all individuals age 25 in 1980, who have some college but lack a four-year college degree. More importantly for our research, the results presented below are not sensitive to the precise value of the dropout rate. Note that the model generates an increasing college dropout rate over time, which is a characteristic that the literature agrees on (for example, Bailey and Dynarski, 2011; Bound et al., 2010, both show a rise in college dropout rates).

- $\hat{\mu}_\epsilon$, σ_ϵ, σ_ζ are determined by the average general ability of college graduates (NLSY79, Armed Forces Qualification Test (AFQT)), the correlation of high school GPA and freshman college GPA of 0.4 (Rothstein, 2004), and the average ability of non-college workers (NLSY79, AFQT). That is, $\overline{h}$ provides a clear dropout cutoff, with overconfidence, $\hat{\mu}_\epsilon$, contributing to the dropout rate, i.e., people attempt college who cannot graduate. Uncertainty over actual abilities is generated by σ_ϵ, with a large literature suggesting that SAT scores and high school performance are an imperfect measure of college performance, and significant information about own ability being revealed through studying at a college level.[10] Lastly, σ_ζ generates imperfect sorting, since individuals have idiosyncratic preferences for school, which translates into a larger variance in the ability of college graduates. This variance is used to explain differences in the average ability between college and non-college individuals.

- σ_θ and δ_1 are determined through three 1980 relative log wage targets: NC90-NC10, C90-C50, and C90-C10. These second moments cover the main intra-group inequality measures that we believe are important. Note that the model has an extra wage target. This additional second moment will provide information on the relevant parameters determining variance: σ_θ, σ_ϵ and σ_ζ. Thus, this extra wage target is important in matching both first and second moments when analyzing wage inequality in the model.

3.4.3 Firm-Specific Parameters

There are four parameters associated with the firm that must be pinned down in 1980, along with two time trends.

Time invariant firm parameters. Two parameters are set outside the estimation procedure. More precisely, the parameter ν is set within the range of standard estimate for college to non-college labor elasticities (see Autor et al., 2008, 1998), and the share parameter on human capital is normalized, $\lambda = 0.5$.

[10]Stinebrickner and Stinebrickner (2013) find that 45 percent of the college dropout rate at Berea College (a small liberal arts college in Kentucky) can be explained by students learning their academic performance in the first two years of college.

The parameters α and ρ are pinned down by matching the share of college graduates (age 25 to 30) in 1980 and the college wage premium in 1980. The resulting elasticity parameter is in line with estimates in Appendix C.2, $\rho = 0.707$.[11]

Time trends. Given the definition of SBTC and MBTC, we normalize A and M to one in 1980. By definition, a rise in A_t will affect both the returns to college ability and math equally, but a rise in M_t will only increase the returns to math. We restrict the growth rate of SBTC such that $\gamma_{a,t} \in [0.018, 0.028]$, which follows from the range of estimates found in Table 2 of Autor et al. (2008). The two growth rates are then calibrated to match the rise in the share of college graduates from 1980 to 2010, along with the rise in the college wage premium. This process yields a SBTC growth rate of $\gamma_a = 0.027$ per annum, which lies at the upper range of possible estimates. As relatively high SBTC decreases the effect of MBTC (Section 3.3.4), the SBTC growth rate estimate is conservative. The calibration suggests that MBTC is substantial during this time period, with $\gamma_m = 0.043$. We present a counterfactual is Section 3.5.1 to understand how important this precise value of γ_m is for the model.

3.4.4 Calibration Summary

Table 3.2 summarizes the estimated and calibrated parameters, with estimated parameters above the center line and calibrated parameters at the bottom.

The 1980 data targets used in pinning down the calibrated parameters are summarized in Table 3.3. The model does well in matching all targets. It only slightly overpredicts the average ability of college graduates and the C90-C50 wage differential of college graduates. For the time trends, the model is unable to match the full rise in the share of college graduates by 2010. However, the model is able to match the share of new college graduates in 2010. This discrepancy can be explained by the draft during the Vietnam War generating above average college graduation rates (Lemieux and Card, 2001).

[11] Alternatively, using the ACS data, but instead computing elasticities across occupations rather than time, as in Appendix C.2, yields similar results as the calibration, with the interval of plus/minus one standard suggesting $\rho \in [0.33, 0.72]$. However, since this method is subject to various assumptions related to the computation of relative wage returns, efficiency units and grouping of occupations, our preferred estimate is using the calibrated elasticity.

Table 3.2: Calibration Summary

Parameter	Value	Source / Type
β	0.9	standard discounting
N	9	retirement at 63 (5 year periods)
σ_η	0.367	transitory wage luck (Guvenen and Kuruscu, 2010)
θ^i	$\sim N(0,\sigma_\theta^2)$	initial ability NLSY79
ν	0.597	elasticity parameter: college to non-college (Autor et al., 2008)
λ	0.5	ability share parameter - normalized
A_{0t}	1.0	SBTC 1980 - normalized
M_{0t}	1.0	MBTC 1980 - normalized
σ_θ	0.142	initial ability
$\hat{\mu}_\epsilon$	0.229	overconfidence
σ_ϵ	0.205	unknown ability component
σ_ζ	0.042	utility of studying
δ_0	-2.953	zero math outcome
δ_1	2.769	math skill slope
$\overline{h}$	0.109	minimum college requirement
$\overline{m}$	0.625	maximum math credits
α	0.411	college share parameter
ρ	0.707	elasticity parameter: ability to math
γ_a	0.027	SBTC growth rate
γ_m	0.043	MBTC growth rate

Table 3.3: Targets Summary

Target	Data (1980)	Model
Fraction 0 Math Credits	0.113	0.113
Fraction 21 Math Credits	0.118	0.118
College Dropout Rate	0.550	0.548
$corr(\theta, h)$	0.400	0.401
$\theta_{college\ graduate}$	0.805	0.821
$\theta_{non-college\ worker}$	-0.282	-0.275
C90-C50	0.568	0.579
C90-C10	1.174	1.154
NC90-NC10	1.104	1.072
College Wage Premium	0.247	0.247
Young College Graduates	0.241	0.247
2010 College Graduates	0.301	0.281
2010 College Wage Premium	0.493	0.495

Table 3.4: US Wage Inequality & Model Results

	Year	Relative Log Wages 90-10	90-50	50-10
All				
Data	1980	116	55	62
	2010	150	77	73
Model	1980	112	55	57
	2010	126	67	59
% Explained		42	52	23
College				
Data	1980	117*	57*	61
	2010	145	74	72
Model	1980	118*	59*	58
	2010	144	71	73
% Explained		92	68	131

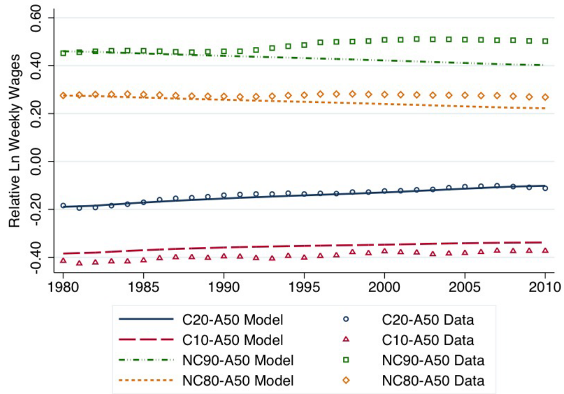
Notes: * Moment targeted in the calibration.

3.5 Results

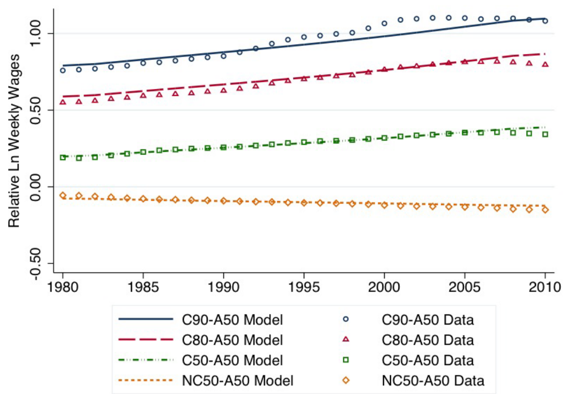
The model accurately captures a variety of inequality dynamics, including many of the general wage trends and all of the intra-college group wage decomposition between 1980 and 2010. These results are driven by the introduction of MBTC into a standard SBTC framework, with the importance of MBTC highlighted through the counterfactual presented in Section 3.5.1. Table 3.4 compares the total US inequality trends and the modeled results. The base model explains nearly half of the rise in inequality of the aggregate US male population.[12]

The model's strength lies in explaining intra-college income inequality, given that MBTC only impacts educated individuals. The model explains almost all the trends for college graduates, both at the top and bottom. At the other end of the income distribution, none of the aggregate non-college trends (NC90-NC10, NC-90-NC50 and NC50-NC10) are well explained. Figure 3.11 shows that the model matches the NC80-A50 and NC50-A50 wage evolution, but it predicts a slight fall in both the NC90-A50 and only a small fall in the NC20-A50 and NC10-A50 percentile (not pictured). In contrast, the data shows a mild increase in the NC90-A50 relative wage and a large decrease in the NC20-A50 and NC10-A50 relative wage. That is, the model is able to replicate the average uneducated worker's wage, but not the extremes. These unmatched trends are unsurprising, as the model ignores the human capital accumulation

[12] The rise in the fraction of college graduates and the college wage premium are matched by construction.



(a) Bottom



(b) Top

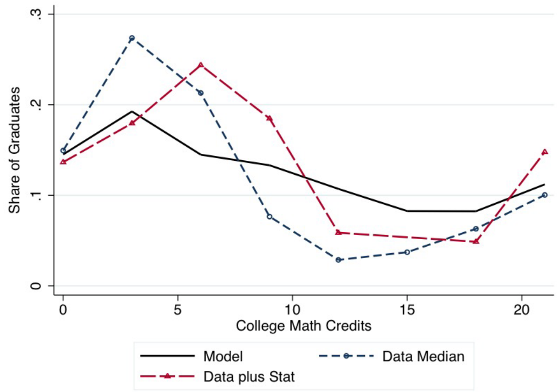
Figure 3.11: Model Wage Trend Decomposition

of uneducated individuals (e.g., unemployment, dropping out of high school or completing vocational 2-year college programs) and MBTC only affects college labor directly. Therefore, the model is more suited to predict income trends within the top half of the distribution, which includes all college graduates. Both Figures 3.11a and 3.11b show an almost perfect matching of the college distribution across the top (C90-A50, C80-A50), median (C50-A50) and bottom (C20-A50, C10-A50) deciles.

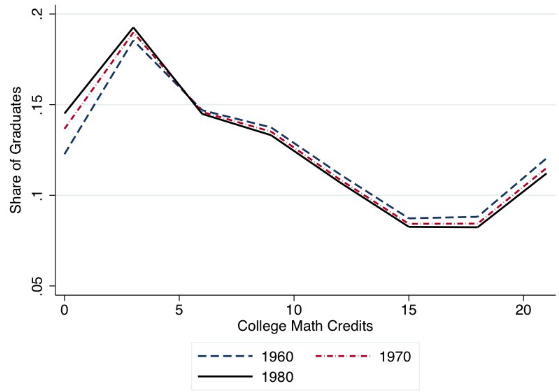
In addition to matching inequality trends, the model also preforms well in matching trends not directly targeted, such as ability distributions by school outcome, dropout rates, etc. Matching these trends is important when assessing the validity of the model and mechanism put forth in this paper. The following paragraphs discuss each trend in detail.

Initial ability. The NLSY79 and NLSY97 show that *ex ante* ability, as measured by the AFQT of college graduates, has fallen from 0.805 to 0.775. Since individuals were aged 12 to 16 in the NLSY97 sample, the natural comparison of college cohorts in the model is 2000-2005. The model slightly overpredicts the average initial ability in 1980 and predicts a fall to 0.769 by 2000 and 0.754 by 2005. While the fall is slightly larger than observed from the NLSY79 to NLSY97, the estimates are close to suggest that the model is not generating the above wage trends through an incorrect composition of college graduates.

Dropout rate. The model generates a rise in the college dropout rate from the matched target of 55 percent in 1980 to 59 percent by 2010. Bound et al. (2010) show that the non-completion rate for males aged 25 went from about 55 percent in 1980 to about 60 percent in 2000. By the year 2000 the model generates a dropout rate of 58 percent, just shy of the data estimates.



(a) 1980 Cohort Data versus Model



(b) Cohort Evolution

Figure 3.12: College Major Graduation Share by Cohorts

College math credits (0 and 21). The model shifts each subsequent cohort towards lower college math levels. In the 1960 cohort 11.3 percent of college graduates (in both the data and model) had zero math credits. By the 1980 cohort this number had risen to 14.9 percent in the data and 14.8 percent in the model. Looking at the fraction of individuals with 21 credits or more, the data falls from 11.8 percent for the 1960 cohort to 10.0 percent for the 1980 cohort. The model replicates just over one-third of this drop, generating a fall from 11.8 to 11.1 percent from 1980 to 2010 for new college graduates.

College majors. Figure 3.12 graphs the share of college graduates over all math outcomes ("college majors"). As math is a continuous variable within the model, Figure 3.12 is computed by scaling all math outcomes such that the maximum math credits earned is 21, and then rounded up to the nearest three credit equivalent. Figure 3.12 (left panel) compares the share of college graduates for the 1980 cohort of the model and data. The model, using a simple math technology, does well in matching the overall shape of college graduates over math outcomes. However, the continuous math outcome variable in the model leads to a smaller mass around three credits compared to the data. It should be noted that a large share of college majors might only require three math credits to graduate, suggesting a kink in the college math "production function." Additionally, reclassifying college credits in statistics as college math credits does not affect the share of individuals with zero math credits, but does generate a larger spread (away from three math credits). Given that the model estimates lie between the two types of math measures (with and without statistics) and the model studies aggregates at the top and bottom of the wage distribution, it is not of first order importance for the results to generate the mass at three credits. Figure 3.12 (right panel) shows the evolution of college graduates by math credits across cohorts in the model. The change

Table 3.5: US Wage Inequality & Constant Skill Supply

	Relative Log Wages		
	90-10	90-50	50-10
College (2010)			
Data	145	74	72
Benchmark	144	71	73
Partial Equilibrium	140	69	71
% Explained Skill Supply	15	17	7

over time in the model mimics the general pattern observed in the US (Figure 3.7).

3.5.1 Counterfactual

We present two counterfactuals: (1) a partial equilibrium exercise, where the graduating cohort attributes remain fixed at the 1960 cohort, and (2) eliminating MBTC by setting $\gamma_{m,t} = 0$.

Supply Effects. Maintaining the attributes of the 1960 cohort for all successive cohorts eliminates the supply effects from the results. Attributes here refer to the distribution of *ex ante* and *ex post* skills. That is, the counterfactual assesses the importance of the falling ability levels of college graduates in driving inequality. Table 3.5 shows how much of the rise in wage inequality in the benchmark is driven by the new "marginal" college graduate being of worse quality (in terms of math skills and general human capital) in 2010 compared to 1980. The aggregate effect is shown in the last row, stating the percentage contribution of changing skill supplies to the rising wage inequality from 1980 to 2010. Of the total wage inequality (90th to 10th percentile college graduate) generated in the benchmark model, 15 percent of the increase in inequality is explained by a deterioration of the skills at the bottom relative to the top graduate from 1980 to 2010. For the 90th to 50th percentile the explanatory power is slightly larger at 17 percent.

MBTC. The removal of the MBTC mechanism clearly reveals the contribution of the returns to acquired math skills. The counterfactual model predicts more individuals attempt college, with a college dropout rate of 60.4 percent in 2010, and more individuals graduate from college, with the share of college graduates increasing to 28.6 percent in 2010 from 28.1 percent in the benchmark model. The share of zero-math credit graduates increases to 15.9 percent and the share of individuals with 21 credits

Table 3.6: US Wage Inequality & Counterfactual Results

	Year	Relative Log Wages 90-10	90-50	50-10
All				
Data	2010	150	77	73
SBTC + MBTC	2010	126	67	59
SBTC	2010	129	68	62
% SBTC Explained		51	55	42
College				
Data	2010	145	74	72
SBTC + MBTC	2010	144	71	73
SBTC	2010	118	60	58
% SBTC Explained		3	4	1

decreases to 10.8 percent by 2010. Simultaneously, the average quality of a college graduate drops from 0.744 to 0.712 in terms of initial ability.

Table 3.6 compares the 2010 wage inequality levels between the data, the benchmark model and the counterfactual model results, omitting the 1980 values, as they are identical to Table 3.4 by construction. At an aggregate level, the counterfactual model is marginally better than the benchmark model in matching broad income inequality trends, particularly at the bottom of the wage distribution. However, these broad measurements hide the intra-college inequality trends that are ignored by the counterfactual model. The second part of Table 3.6 shows that the counterfactual model is unable to match any of the trends in wage inequality between college graduates, explaining only one to four percent of the rise in inequality from 1980 to 2010.

Figure 3.13 further highlights how the counterfactual model matches the aggregated income inequality trends for the wrong reasons. The counterfactual predicts a sharp increase in wages for the bottom college deciles and a sharp fall for wages of the top non-college deciles. This first point is driven by higher returns to the general human capital of college graduates (relative to math). The non-college results are driven by composition. That is, more low ability students enter and graduate from college, effectively decreasing the ability of the top non-college deciles.

The mechanisms of SBTC and MBTC work in opposite directions for the bottom of the college distribution. SBTC alone benefits all college graduates in wage terms. The bottom deciles, in particular, gain compared with the average individual in the economy. Further highlighting the broad power of the SBTC mechanism, the counterfactual pushes the income of the C20 group above the A50 wage, while the C10 wage approaches parity with the A50 wage. In contrast, MBTC is effective at only increase

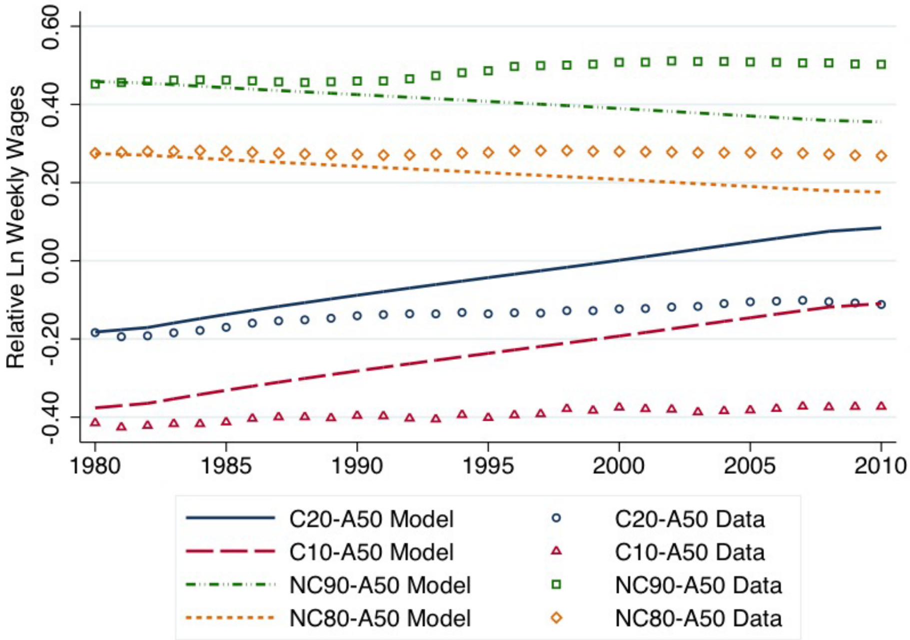


Figure 3.13: US Wage Inequality & Counterfactual Results

the top income deciles, leaving the bottom deciles with decreasing or stagnant wages. Thus, only the combination of these two different technical change concepts can generate the observed wage inequality trends of the US male college graduate population, along with part of the divergence in wages across the entire US male population.

3.6 Conclusion

This paper studies the role of math in determining both inter- and intra-education group wage inequality. The connection between *ex ante* math abilities, college math, and the labor demand for math skills provides a simple and powerful mechanism, explaining a large component of male wage inequality in the US. The estimated structural model highlights the importance of MBTC in explaining both aggregate wage inequality and the wage trends at the extremes of the college distribution. The model is also able to generate average trends for both college and non-college groups, closely matching the general trends achieved by SBTC alone.

Given the results, there are a number of interesting research extensions. As we extend the research on the determinants of wage inequality from a post-education perspective (Huggett et al., 2011; Kambourov and Manovskii, 2009a) to a pre-college education perspective, we assume initial math ability is determined prior to college. Given the central role of math, studying the origins of initial math skills is of primary importance in determining college math outcomes and is a natural first extension of the MBTC mechanism presented here. A second research extension focuses on the gender dimension of education choices, where the specific characteristics of college majors favored by women are assessed. While men optimize their education choices to maximize pecuniary outcomes and choose high-math college majors given initial ability constraints, women exhibit more complex preferences with respect to non-pecuniary outcomes that seem to distort the education decision. In ongoing work, we study women's college decisions in a life-cycle model that accounts for atrophy and repair of skills due to career breaks.

Part III

Appendices

A Appendix: Chapter 1

A.1 Results Appendix

Tables A.1 and A.2 provide the IV-GLS coefficient estimates by broad educational groups.

Table A.1: IV-GLS Estimation Results: Less Than College

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Experience	0.153***	(0.015)	0.125***	(0.013)	0.086***	(0.009)
Experience^2 $\times$ 100	-0.442***	(0.064)	-0.399***	(0.067)	-0.383***	(0.065)
Experience^3 $\times$ 100	0.010***	(0.002)	0.009***	(0.002)	0.009***	(0.002)
Old Job	-0.027**	(0.011)	-0.030***	(0.011)	-0.029***	(0.011)
Firm Tenure	-0.010**	(0.005)	-0.010**	(0.005)	-0.025***	(0.005)
Firm Tenure^2 $\times$ 100	0.004	(0.027)	-0.003	(0.028)	0.044	(0.029)
Career Tenure	0.035***	(0.007)			-0.006	(0.009)
Career Tenure^2 $\times$ 100	-0.293***	(0.075)			-0.048	(0.093)
Career Tenure^3 $\times$ 100	0.008***	(0.003)			0.002	(0.003)
M Tenure			0.012	(0.012)	0.064***	(0.020)
M Tenure^2 $\times$ 100			-0.057	(0.150)	-0.944**	(0.453)
M Tenure^3 $\times$ 100			-0.001	(0.005)	0.049*	(0.028)
V Tenure			0.011	(0.012)	0.009	(0.018)
V Tenure^2 $\times$ 100			-0.150	(0.146)	0.320	(0.381)
V Tenure^3 $\times$ 100			0.006	(0.005)	-0.013	(0.022)
S Tenure			0.022*	(0.012)	0.048**	(0.020)
S Tenure^2 $\times$ 100			-0.198	(0.142)	-1.192**	(0.487)
S Tenure^3 $\times$ 100			0.004	(0.005)	0.044	(0.033)
T Tenure			-0.001	(0.012)	0.047***	(0.014)
T Tenure^2 $\times$ 100			0.042	(0.141)	-0.280	(0.268)
T Tenure^3 $\times$ 100			0.001	(0.005)	0.009	(0.014)
Observations	17,339		17,316		17,274	
Individuals	1,426		1,426		1,424	

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See notes Table 1.4 and text for details.

Table A.2: IV-GLS Estimation Results: College

VARIABLES	Base (1)		Skill (2)		Skill Equivalent (3)	
Experience	0.048**	(0.023)	0.027	(0.031)	0.040**	(0.018)
Experience^2 × 100	-0.146	(0.117)	-0.046	(0.119)	-0.021	(0.117)
Experience^3 × 100	0.001	(0.004)	-0.002	(0.004)	-0.002	(0.004)
Old Job	-0.030*	(0.017)	-0.042**	(0.018)	-0.038**	(0.018)
Firm Tenure	-0.016**	(0.008)	-0.020**	(0.009)	-0.029***	(0.009)
Firm Tenure^2 × 100	0.028	(0.045)	0.052	(0.051)	0.076	(0.049)
Career Tenure	0.071***	(0.012)			0.010	(0.018)
Career Tenure^2 × 100	-0.581***	(0.142)			-0.095	(0.194)
Career Tenure^3 × 100	0.014***	(0.005)			0.001	(0.007)
M Tenure			0.024	(0.021)	0.049	(0.030)
M Tenure^2 × 100			-0.132	(0.272)	-0.443	(0.558)
M Tenure^3 × 100			0.003	(0.010)	0.021	(0.030)
V Tenure			0.028	(0.023)	0.079***	(0.029)
V Tenure^2 × 100			-0.315	(0.297)	-1.396***	(0.522)
V Tenure^3 × 100			0.007	(0.011)	0.062**	(0.027)
S Tenure			0.020	(0.024)	-0.005	(0.035)
S Tenure^2 × 100			-0.052	(0.328)	0.603	(0.722)
S Tenure^3 × 100			0.002	(0.013)	-0.034	(0.042)
T Tenure			0.023	(0.023)	0.042	(0.029)
T Tenure^2 × 100			-0.342	(0.305)	-0.674	(0.514)
T Tenure^3 × 100			0.010	(0.012)	0.016	(0.027)
Observations	6,752		6,733		6,655	
Individuals	626		626		624	

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See notes Table 1.4 and text for details.

B Appendix: Chapter 2

B.1 National Longitudinal Survey of Youth 1979 (NLSY)

The NLSY is a nationally representative sample of individuals aged 14 to 22 in 1979. Surveys were conducted on an annual basis until 1994 and biannually thereafter. The original sample included 12,686 men and women.

Wage information is reported at the survey dates, and is adjusted to constant 2000 US Dollars. Survey observations without wage data are dropped from the sample, as are those without occupation information. We also drop individuals with military occupations as-of the interview date because their wage observation may not be determined by general labor market forces.

The NLSY sample provides weekly observations for employment status from which career breaks are constructed. Thus, each observation in the data set has two measures of employment gaps in weeks: (1) cumulative length of all gaps; and (2) length of prior gap. These gap measures account for employment status values in a conservative manner. I.e., the six labor force status values (e.g., unemployed, active military service) used when an occupational code is not provided are considered unemployment spells. This means that the number and length of gaps is likely overestimated, reducing the effect of each gap on wages. The reason military service is considered a work gap concerns how employers view this experience. If the tasks performed while undertaking military service are relevant to the formal labor market, then military service could be considered employment. However, it is not clear how relevant military service tasks are to employers, and coding these values as unemployment is a conservative assumption. Note that individuals employed full-time within military service were dropped prior to the employment gap variable construction, leaving only individuals with short-term military service.

After accounting for missing and inconsistent information, the data set contains individual-level observations across time for wages, occupation, employment gap measures and multiple individual characteristics, such as gender and education. Thus, the final sample contains 5,652 individuals, of which 2,782 (49 percent) are males.

B.2 Occupational Information Network (O*net) and Armed Services Vocational Aptitude Battery (ASVAB)

The Occupational Information Network (O*net) database contains detailed descriptive information for more than 900 occupations, and succeeds the Dictionary of Occupational Titles (DOT). Whereas the DOT is based on direct expert observations of occupations, the O*net sends questionnaires to a random sample of workers based on their occupations. Each worker completes one-quarter of the questions, which are organized into eight broad categories. Three categories are of particular interest:

- Knowledge: Biology, Building and Construction, Chemistry, Computers and Electronics, Engineering and Technology, English Language, Mathematics, Mechanical, Physics
- Skill: Equipment Maintenance, Equipment Selection, Installation, Mathematics, Operation and Control, Reading Comprehension, Repairing, Science, Technology Design
- Ability: Trouble Shooting, Deductive Reasoning, Inductive Reasoning, Information Ordering, Mathematical Reasoning, Number Facility, Oral Comprehension, Written Comprehension

Besides recording standard survey questions regarding family status and work, the NLSY respondents took the Armed Services Vocational Aptitude Battery (ASVAB) in the Summer and Fall of 1980, which was administered by the US Departments of Defense and Military Services. The ASVAB was designed to provide high school graduates with better career guidance compared to a simple general or academic ability test. The test components can be grouped into four major skill types/components:

1. Math is composed of “Arithmetic Reasoning” and “Mathematics Knowledge.”
2. Verbal is composed of “Word Knowledge” and “Paragraph Comprehension.”
3. Technical is composed of “Auto and Shop, Mechanical Comprehension” and “Electronics Information.”
4. Science is composed of “General Science Knowledge.”

In an effort to make career matching easier for new high school graduates, the ASVAB Career Exploration Program decided to match occupational information from O*net data to the ASVAB test components. For this purpose, 26 occupational descriptors of the O*net were matched to the ASVAB test sections listed above. The descriptors include information of knowledge, skill and ability required in preforming each O*net occupation. As the list of O*net descriptors above reveals, each has a natural mapping into math, verbal, technical and science skill components. The mapping to four ASVAB components was determined by experts using a six-point scale ranging from "Highly related" to "Not at all related." Experts came from the field of industrial/organizational psychology, general psychology, and psychometrics.

B.3 Results Appendix: Depreciation Rates

Table B.1 includes part-time workers. The results are similar in sign and magnitude to the results for full-time workers only (see Table 2.3).

Table B.2 provides GLS results for the base regression outlined in Table 2.3. The results are similar in sign and magnitude to the OLS estimates for full-time workers.

Table B.3 shows depreciation rates for male and female workers using GLS to account for serially correlated errors. The coefficients are slightly smaller for college educated women compared to OLS. However, the general hypothesis still holds, with high math occupations experiencing larger wage penalties for gaps and high verbal (and science) occupations off-setting some of the penalty. In addition, the results now also suggest a penalty in highly technical occupations. For non-college women the skill-specific depreciation rates are now statistically significant, albeit still smaller than those for college graduates.

Table B.1: Depreciation Rates

VARIABLES	LTC (1)	C+ (2)	LTC (3)	C+ (4)	LTC (5)	C+ (6)
Part-time	-0.097***	-0.146***	-0.070***	-0.103***	-0.069***	-0.103***
	(0.007)	(0.014)	(0.007)	(0.015)	(0.007)	(0.015)
Math	-0.084*	0.400***	-0.097**	0.367***	-0.061	0.442***
	(0.047)	(0.111)	(0.048)	(0.113)	(0.049)	(0.116)
Verbal	-0.012	0.631***	-0.004	0.686***	-0.027	0.628***
	(0.040)	(0.095)	(0.042)	(0.095)	(0.043)	(0.098)
Science	-0.388***	-0.904***	-0.392***	-0.866***	-0.406***	-0.949***
	(0.051)	(0.155)	(0.054)	(0.154)	(0.054)	(0.160)
Technical	0.599***	0.376***	0.613***	0.375***	0.597***	0.377***
	(0.037)	(0.112)	(0.038)	(0.114)	(0.039)	(0.118)
Cumm Gap	0.347***	0.484***			0.353***	0.487***
	(0.033)	(0.067)			(0.033)	(0.067)
Last Gap			-0.402***	-0.664***	-0.398***	-0.629***
			(0.049)	(0.167)	(0.049)	(0.167)
Cumm Gap M	-0.068***	-0.091***			-0.066***	-0.085***
	(0.014)	(0.021)			(0.014)	(0.021)
Last Gap M			-0.670**	-1.291**	-0.473	-1.092*
			(0.290)	(0.641)	(0.293)	(0.633)
Cumm Gap V	0.044***	0.050**			0.041***	0.049**
	(0.011)	(0.022)			(0.011)	(0.021)
Last Gap V			0.316	0.121	0.208	0.014
			(0.249)	(0.593)	(0.250)	(0.591)
Cumm Gap S	0.048***	0.094***			0.047***	0.091***
	(0.013)	(0.027)			(0.013)	(0.027)
Last Gap S			0.521*	1.245	0.369	1.006
			(0.288)	(0.771)	(0.287)	(0.764)
Cumm Gap T	-0.002	-0.010			0.000	-0.013
	(0.010)	(0.022)			(0.010)	(0.022)
Last Gap T			0.095	0.133	0.107	0.210
			(0.206)	(0.687)	(0.209)	(0.687)
Observations	47,862	16,353	47,862	16,353	47,862	16,353
R-squared	0.328	0.348	0.327	0.346	0.330	0.351

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Robust standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See Table 2.3 for further details.

Table B.2: Depreciation Rates (GLS)

VARIABLES	LTC (1)	C+ (2)	LTC (3)	C+ (4)	LTC (5)	C+ (6)
Math	-0.074***	-0.052***	-0.074***	-0.115***	-0.049***	-0.055***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Verbal	-0.151***	0.302***	-0.145***	0.281***	-0.169***	0.254***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Science	-0.137***	-0.270***	-0.139***	-0.220***	-0.156***	-0.258***
	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
Technical	0.352***	0.208***	0.355***	0.262***	0.351***	0.224***
	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
Cumm Gap	0.042***	0.101***			0.051***	0.107***
	(0.000)	(0.001)			(0.000)	(0.001)
Last Gap			-0.545***	-0.980***	-0.531***	-0.966***
			(0.001)	(0.003)	(0.001)	(0.003)
Cumm Gap M	-0.038***	-0.057***			-0.035***	-0.055***
	(0.000)	(0.000)			(0.000)	(0.000)
Last Gap M			-1.070***	-0.418***	-1.034***	-0.339***
			(0.006)	(0.010)	(0.006)	(0.010)
Cumm Gap V	0.038***	0.020***			0.036***	0.016***
	(0.000)	(0.000)			(0.000)	(0.000)
Last Gap V			0.338***	1.723***	0.300***	1.708***
			(0.005)	(0.009)	(0.005)	(0.009)
Cumm Gap S	0.035***	0.041***			0.031***	0.040***
	(0.000)	(0.000)			(0.000)	(0.000)
Last Gap S			1.129***	-0.365***	1.068***	-0.424***
			(0.005)	(0.013)	(0.006)	(0.013)
Cumm Gap T	0.000	0.035***			0.003***	0.038***
	(0.000)	(0.000)			(0.000)	(0.000)
Last Gap T			-0.008**	-0.376***	-0.007*	-0.438***
			(0.004)	(0.011)	(0.004)	(0.011)
Observations	40,411	14,345	40,411	14,345	40,411	14,345
Individuals	3,796	1,385	3,796	1,385	3,796	1,385

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: Standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See Table 2.3 for further details.

Table B.3: Gender-specific Depreciation Rates (GLS)

	Male		Female	
VARIABLES	LTC (1)	C+ (2)	LTC (3)	C+ (4)
Math	-0.061***	-0.098***	0.010***	-0.006***
	(0.001)	(0.002)	(0.001)	(0.002)
Verbal	-0.293***	-0.019***	0.014***	0.563***
	(0.001)	(0.002)	(0.001)	(0.002)
Science	-0.121***	-0.269***	-0.185***	-0.504***
	(0.001)	(0.003)	(0.001)	(0.003)
Technical	0.348***	0.370***	0.252***	0.404***
	(0.001)	(0.002)	(0.001)	(0.002)
Cumm Gap	0.014***	0.046***	0.132***	0.203***
	(0.001)	(0.001)	(0.001)	(0.001)
Last Gap	-0.631***	-1.026***	-0.490***	-0.768***
	(0.001)	(0.004)	(0.001)	(0.004)
Cumm Gap M	-0.009***	-0.022***	-0.056***	-0.043***
	(0.000)	(0.001)	(0.000)	(0.001)
Last Gap M	-1.400***	1.001***	-0.417***	-1.777***
	(0.007)	(0.015)	(0.009)	(0.014)
Cumm Gap V	0.035***	0.017***	-0.001***	0.025***
	(0.000)	(0.001)	(0.000)	(0.001)
Last Gap V	0.041***	2.402***	0.041***	1.224***
	(0.007)	(0.015)	(0.008)	(0.013)
Cumm Gap S	-0.014***	-0.020***	0.088***	0.087***
	(0.000)	(0.001)	(0.000)	(0.001)
Last Gap S	1.430***	-2.151***	0.608***	1.323***
	(0.007)	(0.020)	(0.009)	(0.015)
Cumm Gap T	0.036***	0.074***	-0.004***	-0.058***
	(0.000)	(0.001)	(0.000)	(0.001)
Last Gap T	0.219***	-0.513***	0.389***	-0.654***
	(0.005)	(0.017)	(0.010)	(0.017)
Observations	22,663	7,876	17,748	6,469
Individuals	1,872	671	1,924	714

Statistical significance levels: *** p<0.01, ** p<0.05, * p<0.1

Notes: Robust standard errors in parentheses.

Math (M), verbal (V), science (S) and technical (T) variables are denoted by their respective first letter.

See Table 2.3 for further details.

C Appendix: Chapter 3

C.1 Data Appendix: Aptitude Measures

This appendix lists the definition of the three DOT variables referenced in the paper. The three aptitude variables used are from the DOT 1977 and 1991 editions. We are unable to use the DOT successor, the Occupational Information Network (O*net), due to a measurement discontinuity in the recorded aptitudes.

In general, aptitudes measure the ability an individual must possess in order to perform a job successfully. More precisely, the measure is a function of the share in the population that meets this ability level. That is, there are five categories: (1) the bottom 10 percent of the population, (2) the bottom third excluding the bottom 10 percent, (3) the middle third, (4) the top third excluding the top 10 percent, and (5) the top 10 percent. We translate these measures to a scale ranging from zero to one, e.g., an aptitude above 0.66 would correspond to an individual in the top-third of the population.

Out of the 11 measures reported in the DOT 1977 and 1991, we use the three measures: general, numerical and verbal ability.

- General ability is the ability to understand instructions, understand principles and to make judgments. It encompasses a number of skills, e.g., using logic and scientific thinking, understanding procedures, establishing facts and drawing conclusions, etc. This measure is highly correlated with the ability to perform well in school.

- Numerical aptitude is the ability to perform arithmetic. The complexity and speed of operations is taken into account when assigning the category.

- Verbal aptitude is the ability to understand and use language effectively. Both oral and written skills, including the use of technical terminology, are taken into account when assigning categories for each occupation.

C.2 Empirical Appendix: Estimating MBTC Over Time

Figure 3.4 graphs the relative wage rates between high-math college and non-college occupations, and low-math college and non-college occupations. Relative wages are normalized to zero in 1974 for easy comparison of the two MBTC and SBTC trends.

Relative labor supplies and, consequently, relative wages rates, are computed following Hansen (1993) in estimating labor efficiency units at time t as,

$$L_{jt}^{E} = \sum_{k} \psi_k L_{jt,k}, \tag{C.1}$$

where $L_{jt,k}$ is the total labor supply of group k of labor type $j = \{u, e, me\}$ (non-college (u), low-math college labor (e) and high-math labor (me)), and ψ_k is the group's weight. Weights are determined by,

$$\psi_k = \frac{\overline{\omega}_k}{\overline{\omega}}, \tag{C.2}$$

the average log weekly wage of group k over the average wage of the entire population (across individuals over the entire time period). Groups are made up of a given five-year birth cohort, sex and education group (high school dropout, high school graduate, some college, college graduate, and post-graduate).

Using this definition of efficiency units of labor, log relative wage rates are,

$$ln(w_{jt}) - ln(w_{ut}) = \sum_{k} f_{jt,k} \frac{\omega_{jt,k}}{L_{jt}^{E}} - \sum_{k} f_{ut,k} \frac{\omega_{ut,k}}{L_{ut}^{E}} \; for\; j = e, me, \tag{C.3}$$

where $f_{jt,k}$ is the fraction of group k of labor type j individuals in the economy each period.

C.2.1 Quantifying MBTC versus SBTC

Given wage rates and relative labor supplies, we can quantify the difference between SBTC and MBTC as captured in Figure 3.4.

Analogous to the firms problem in Section 3.3, we define a nested-CES between the three types of labor $j = \{u, e, me\}$,

$$Y_t = \left[\alpha L_{ut}^{\nu} + (1-\alpha)\left[\lambda \left(A_t L_{et}\right)^{\rho} + (1-\lambda)\left(A_t M_t L_{met}\right)^{\rho}\right]^{\frac{\nu}{\rho}}\right]^{\frac{1}{\nu}}, \tag{C.4}$$

with SBTC (labor augmenting technology for all college graduates equally) as, $A_t = (1+\gamma_{at})A_{t-1}$ and MBTC (labor augmenting technology for high-math labor only) as, $M_t = (1+\gamma_{mt})M_{t-1}$.

From the firm's cost minimization problem, we obtain two relative wage equations,

$$\ln\left(\frac{w_{et}}{w_{ut}}\right) \approx C + (\nu-1)\ln\left(\frac{L_{et}}{L_{ut}}\right) + \nu\ln\left(A_t\right) + \frac{1-\lambda}{\lambda}M_t^{\rho}\left(\frac{\nu-\rho}{\rho}\right)\left(\frac{L_{met}}{L_{et}}\right)^{\rho} \quad \text{(C.5)}$$

and

$$\ln\left(\frac{w_{met}}{w_{ut}}\right) \approx C + (\nu-1)\ln\left(\frac{L_{met}}{L_{ut}}\right) + \nu\ln\left(A_t\right) + \nu\ln\left(M_t\right) + \frac{\lambda}{1-\lambda}\left(\frac{1}{M_t}\right)^{\rho}\left(\frac{\nu-\rho}{\rho}\right)\left(\frac{L_{et}}{L_{met}}\right)^{\rho}. \quad \text{(C.6)}$$

Equation (C.5) shows the college premium of low-math college graduates relative to non-college labor, and Equation (C.6) shows the relationship between high-math college graduate wages and non-college wage returns. As in Krusell et al. (2000), we can analyze the growth in relative wages using these two equations, assuming $\lambda = 1-\lambda$,

$$g_{w_{et}} - g_{w_{ut}} = (\rho-1)g_{L_{et}} - (\nu-1)g_{L_{ut}} + \nu g_{A_t} + (\nu-\rho)\left(g_{L_{met}} + g_{M_t}\right) \quad \text{(C.7)}$$

and

$$g_{w_{met}} - g_{w_{ut}} = (\rho-1)g_{L_{met}} - (\nu-1)g_{L_{ut}} + \nu g_{A_t} + \rho g_{M_t} + (\nu-\rho)g_{L_{et}}. \quad \text{(C.8)}$$

With the two Equations (C.7) and (C.8) we can compute the two unknowns of interest, $g_{A_t} = \gamma_{at}$ and $g_{M_t} = \gamma_{mt}$.

Having computed efficiency units of labor and relative wages rates from the CPS 1974 to 2010, all that remains is pinning down the elasticities between college and non-college labor, $\frac{1}{1-\nu}$, and between low- and high-math labor, $\frac{1}{1-\rho}$. The parameter $\nu = 0.597$ is set as in the simulation (see Section 3.4). However, ρ used here is not directly comparable with the parameter from Section 3.3. Therefore, in the appendix we estimate ρ using CPS data.

The firm's nested-CES minimization problem provides the following relative wage equation,

$$\ln(w_{met}) - \ln(w_{et}) = \ln\left(\frac{(1-\lambda)}{\lambda}\right) + \rho\ln(M_t) + (\rho-1)\ln\left(\frac{L_{met}}{L_{het}}\right). \quad \text{(C.9)}$$

Assuming a linear time trend for $\ln(M_t)$, we can estimate this equation using relative wages and efficiency units of labor to obtain ρ. Table C.1 summarizes the results.

Table C.1: College-Labor CES

Variable	Coefficient	(Std. Err.)
Time	0.002***	(0.000)
Labor Supply	-0.274**	(0.104)
Intercept	-0.035	(0.066)
N		37
R^2		0.821

Statistical significance levels: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

The parameter $\rho = 0.726$ is significant at five percent.[1] In contrast, the equivalent regression for non-college labor suggests perfect substitution between low- and high-math uneducated labor. This is also consistent with the findings of Figure 3.2b.

With the elasticity parameters, $\rho = 0.726$ and $\nu = 0.597$, we can compute the growth in A_t and M_t to be consistent with the growth in relative wages of non-college/college and low-/high-math labor, given the growth in efficiency units from the CPS during 1974 to 2010.

Using the whole time period, this simple accounting exercise suggests that $\gamma_{et} = 0.021$ and $\gamma_{met} = 0.003$. That is, MBTC is positive and larger than zero. To put this value into perspective, comparing A_t versus $A_t \times M_t$, the growth rates suggest that labor augmenting technical change on high-math occupations grew about 16 percent per annum faster than on low-math occupations.

For robustness, using ρ of plus/minus one standard deviation, $\rho = 0.621$ and $\rho = 0.830$, the relative growth of $A_t \times M_t$ is 14 and 17 percent larger, respectively. The more complimentary the two types of college labor, the smaller MBTC needs to be to match the relative wage growth of both low- and high-math labor to non-college labor.

[1]The elasticity is not sensitive to the precise partitioning of low- and high-math occupations. For example, splitting the sample by the top-third versus bottom two-third yields similar results.

Part IV

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